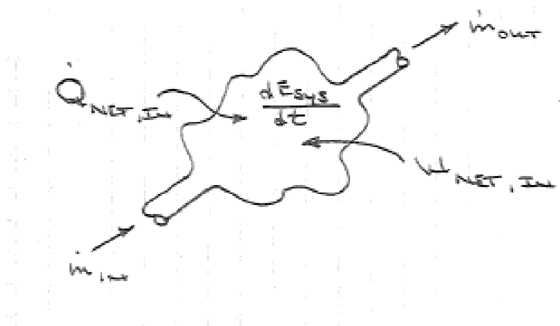
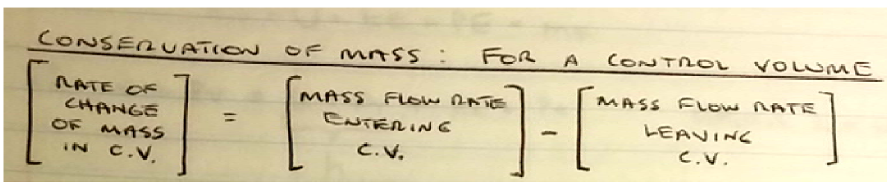
Control Volume Analysis

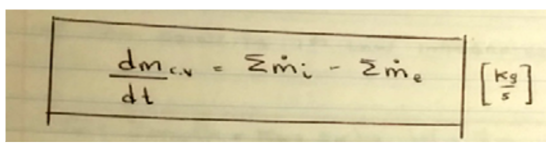
So far, our derivation of the energy balance equation has been limited to closed systems.

Let us now develop a generalized energy balance for open systems or Control Volumes.



In open systems, there is mass flow across the control volume boundary.





Since energy is conserved across the control volume, we must account for the energy accompanying the mass.

Total mass in the control volume at an instant is

$$m_{cv}(t) = \int_V \rho dV$$

Where:

- v_n = component of the relative velocity normal to dA in the direction of flow

Mass flow rate, $\dot{m}$, entering or exiting the control volume is

$$\begin{bmatrix} amount\ of\ mass\ crossing\ dA \\ during\ the\ time\ interval\ \Delta T \end{bmatrix} = \rho(v_n \Delta T)dA$$

$$\rho(v_n \Delta T)dA = \left[\frac{kg}{m^3}\right]\left[\frac{m}{s}\right][s][m^2] = [kg]$$

Divide both sides by ΔT

$$\begin{bmatrix} instantaneous\ mass\ flow \\ rate\ across\ dA \end{bmatrix} = \rho v_n dA$$

Integrate across all of area, A

$$\dot{m} = \int_A \rho v_n dA$$

In one dimensional flow: $\dot{m} = \rho A v = \frac{Av}{v}$

Conservation of Energy for a Control Volume

$$\begin{bmatrix} Change\ of\ energy \\ in\ CV \end{bmatrix} = \begin{bmatrix} Energy \\ entering\ CV \end{bmatrix} - \begin{bmatrix} Energy \\ leaving\ CV \end{bmatrix}$$

$$\Delta(u + KE + PE)_{cv} = Q - W + m_i\left(u_i + \frac{v_i^2}{2} + gz_i\right) - m_e(u_e + \frac{v_e^2}{2} + gz_e)$$

Time rate energy balance

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i\left(u_i + \frac{v_i^2}{2} + gz_i\right) - \dot{m}_e(u_e + \frac{v_e^2}{2} + gz_e)$$

Work in a Control Volume

There are two contributions to the work term $\dot{W}$

$$\dot{W} = \dot{W}_p + \dot{W}_{cv}$$

1. $\dot{W}_p$ = Work associated with the fluid pressure as mass is introduced at inlets and removed at exits
2. $\dot{W}_{cv}$ = All other work effects associated with rotating shafts, displacement of the boundary, electrical, magnetic, etc.

Rate of energy transfer by work = force *x* velocity $(W = F \cdot d)$

$$\boldsymbol{flow\ work}\ at\ the\ exit = \begin{bmatrix} Time\ rate\ of\ energy\ transfer \\ by\ work\ \boldsymbol{from}\ the\ control \\ volume\ at\ exit \end{bmatrix} = (p_e A_e)v_e$$

Where:

- p_e is the pressure of the flowing matter at exit

Therefore

$$\dot{W} = \dot{W}_{cv} + (p_e A_e)v_e - (p_i A_i)v_i$$

or since $Av = \dot{m}v$, where v is the specific volume, this can be rewritten:

$$\dot{W} = \dot{W}_{cv} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i)$$

Substituting these new work components into the energy balance yields

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(u_i + p_i v_i + \frac{v_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + p_e v_e + \frac{v_e^2}{2} + gz_e\right)$$

Enthalpy

The sum of the internal energies, u, and flow work. It is often expressed in its per unit mass form, as presented in the energy balance

$$h = u + pv$$

Substituting enthalpy into the energy balance, we get the general form of the 1st law energy balance

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(h_i + \frac{v_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{v_e^2}{2} + gz_e\right)$$

Remember if steady state, $\frac{dE_{cv}}{dt} = 0$, and $\dot{m}_i = \dot{m}_e$,

Therefore the steady state energy balance may be expressed as:

$$\frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_i - h_e) + \frac{v_i^2 - v_e^2}{2} + g(z_i - z_e) = 0$$