ENGR 292 – Fluids and Thermodynamics Midterm Feb.28 2017

Midterm Exam Solution

Section 1 (10 points)

1) A hydraulic cylinder must be able to exert a force of 8700 lb. The piston diameter is 1.5 in. What is the required pressure in the oil? (2 points)
   Solution:

$$P = \frac{F}{A}$$
$$F = 8700\ lb$$
$$D = 1.5\ in$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (1.5 in)^2}{4} = 1.767\ in^2$$
$$P = \frac{F}{A} = \frac{8700 lb}{1.767 in^2} = 4923\ psi$$

2) A certain hydraulic system operates at 3000 psi. Compute the percentage change in the volume of the oil in the system as the pressure in increased from zero to 3000 psi. The Bulk Modulus of the oil is 189000psi. (2 points)
   Solution:

$$\frac{\Delta V}{V} = \frac{-p}{E} = \frac{-3000\text{ psi}}{189000\text{ psi}} = -0.0159 = \mathbf{-1.59\%}$$

3) A cylindrical container is 150 mm in diameter and weighs 2.25 N when empty. When filled to a depth of 200 mm with a certain oil, it weighs 35.4 N. Calculate the specific gravity of the oil. $\gamma_{water@4°C} = 9.81kN/\mathrm{m}^3$ (3 points)
Solution:

$$W_o = 35.4\,N - 2.25N = 33.15\,N$$
$$D = 150mm = 0.15m;\quad h = 200\,mm = 0.2m$$
$$V_o = Ah = \frac{\pi D^2}{4}h = \frac{\pi D^2 h}{4} = \frac{\pi(0.15m)^2(0.2m)}{4} = 3.53\times10^{-3}m^3$$
$$\gamma_o = \frac{W_o}{V_o} = \frac{31.35N}{3.53\times10^{-3}m^3} = 9.39\times10^3 kN/m^3$$
$$sg = \frac{\gamma_o}{\gamma_{w@4°C}} = \frac{9.39kN/m^3}{9.81kN/m^3} = 0.956$$

4) The measurement of shear stress (τ) on a surface and the rate of change in shear strain at the surface for a fluid has been determined by experiment to be $\tau = 0.14N/m^2$; $\Delta\mathrm{v}/\Delta y = 13.63\ s^{-1}$, determine the dynamic viscosity (η) of this fluid. (3 points)
Solution:

$$\eta = \tau/(\Delta\mathrm{v}/\Delta y) = (0.14N/m^2)/(13.63s^{-1})$$
$$= 0.01027\ N/(m^2 s^{-1}) = 0.01027\ Pa.s$$

Section 2 (35 points)

1) A rectangular block of wood, floats with one face horizontal in a fluid (density of the fluid $\rho_{fluid} = 900\ kg/\mathrm{m}^3$). The Wood's density is $\rho_{wood} = 750\ kg/\mathrm{m}^3$. Determine the percentage of the wood, which is not submerged. (5 points)
Solution:

$F_b = V_d \times \gamma_{fluid} = V_d \times \rho_{fluid} \times g$
$W = V \times \rho_{wood} \times g$
In the steady state,
$F_b = W$

Therefore, $V_d \times \rho_{fluid} \times g = V \times \rho_{wood} \times g$

$$\frac{V_d}{V} = \frac{\rho_{wood}}{\rho_{fluid}}$$

Therefore the percentage of the wood which is NOT submerged is:

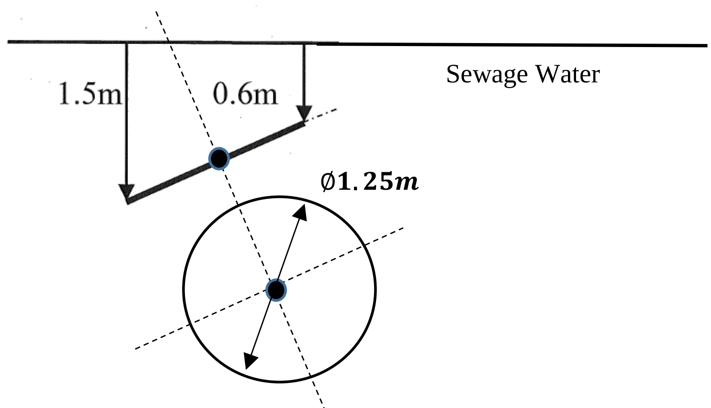
$$(V - V_d)/V = 1 - \frac{V_d}{V} = 1 - \frac{\rho_{wood}}{\rho_{fluid}} = 1 - \frac{750}{900} = 17\%$$

2) How deep can a diver descent in ocean water without damaging his watch, which will withstand an absolute pressure 552 kPa? Take the density of ocean water, $\rho_{\text{ocean water}}$=1025kg/m^3. g=9.81m/s^2. $P_{atm} = 101$kPa. (5 points)
Solution:

As we know: $P_{abs} = \rho gh + P_{atm}$
Hence

$$h = \frac{P_{abs} - P_{atm}}{\rho g} = \frac{(552 - 101) \times 1000Pa}{(1025\text{kg}/m^3) \times (9.81\text{m}/s^2)}$$
$$= \frac{(552 - 101) \times 1000N/m^2}{(1025\text{kg}/m^3) \times (9.81\text{m}/s^2)}$$
$$= \frac{(552 - 101) \times 1000kg\text{m}/s^2/m^2}{(1025\text{kg}/m^3) \times (9.81\text{m}/s^2)}$$
$$= 44.85 \text{ m}$$
$$h = 44.85\ m \text{ or } h = 147\ ft$$

3) A flat circular plate, 1.25 m diameter is immersed in sewage water (density $\rho_{\text{sewage water}}$= 1200 kg/m^3) such that its greatest and least depths are 1.5 m and 0.6 m respectively. Determine the force exerted on one face by the sewage water pressure. g=9.81m/s^2. (10 points)



Solution:

Area of the flat circular plate:

$$A = \frac{\pi D^2}{4} = \frac{\pi 1.25^2}{4} = 1.228\ m^2$$

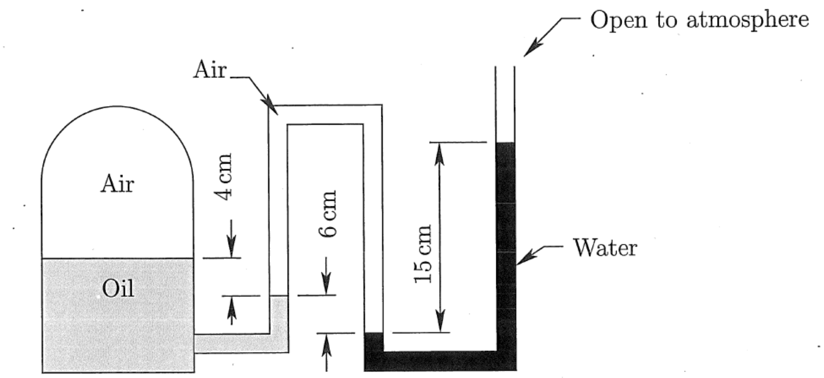
Depth to the centroid:

$$h_c = \frac{1.5 + 0.6}{2} = 1.05\ m$$

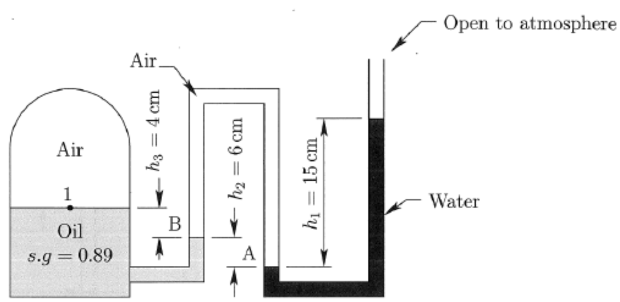
Resultant Force:

$$F_R = \rho g A h_c = 1200 \times 9.81 \times 1.228 \times 1.05 = 15180\ N$$

4) The tank shown holds oil (density ρ_{oil} = 890 kg/m^3). The top of the tank is closed and the space in the tank above the oil contains air. The U-tube manometer contains water and the displacements are as indicated. What is the pressure of the air in the tank? ρ_{water}= 998kg/m^3; the density of the air can be neglected. g=9.81m/s^2 (10 points)



Solution:



$$h_1 = 0.15m$$
$$h_2 = 0.06m$$
$$h_3 = 0.04m$$

$$\rho_{water} = 1000\text{kg/m}^3$$
$$\rho_{oil} = sg \times \rho_{w@4°C} = 0.89 \times 1000\text{kg/m}^3 = 890\text{kg/m}^3$$

Since the density of the air is negligible, $\rho_{air} = 0$; Use gage pressures.

$$P_1 = h_1\rho_{water}g - h_2\rho_{air}g - h_3\rho_{oil}g = (h_1\rho_{water} - h_2\rho_{air} - h_3\rho_{oil})g$$

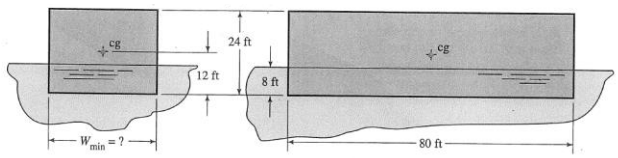
$$= ((0.15m) \times (1000\text{kg/m}^3) - 0 - (0.04m)(890\text{kg/m}^3))(9.81\text{m/s}^2)$$

$$= (150\text{kg/m}^2 - 35.6\text{kg/m}^2)(9.81\text{m/s}^2)\text{=1122.26 Pa=1.12 kPa}$$

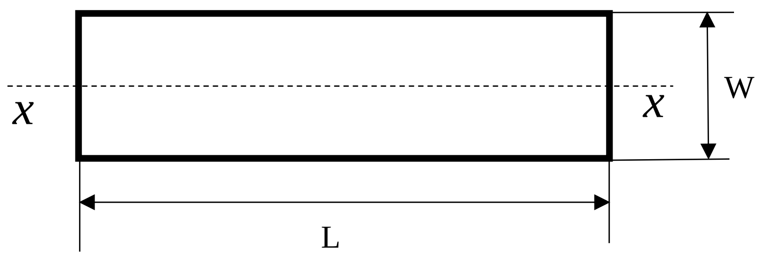
$$P_{1gage} = 1.12kpa$$

$$P_{1abs} = 1.12kpa + 101kpa = 102.22\ kpa$$

5) Figure below shows a river scow (Length L=80ft, Height H=24 ft) used to carry bulk materials. Assume that the scow's center of gravity is at its centroid and that it floats with 8.00 ft submerged. Determine the minimum width that will ensure stability in water. (5 points)



Moment of Inertia of rectangular area about *x-x* axis: $I_{xx} = \frac{LW^3}{12}$



Solution:

We use the bottom of the river scow as the reference.
The height of the river scow is given:

$$H = 24\ ft$$

Therefore, center of gravity

$$y_{cg} = \frac{H}{2} = 12ft$$

Draft of the river scow is given:

$$X = 8\ ft$$

Therefore, center of buoyancy

$$y_{cb} = \frac{X}{2} = 4ft$$

$$y_{mc} = y_{cb} + MB$$

In order to ensure the stability of the river scow, then $y_{mc} > y_{cg}$ ➔ $y_{cb} + MB > y_{cg}$

Therefore, $MB > y_{cg} - y_{cb} = 12ft - 4ft = 8ft$

$$MB_{min} = 8ft$$

$$MB = \frac{I}{V_d} = \frac{LW^3/12}{LWX} = \frac{W^2}{12X}$$

Therefore,

$$MB_{min} = \frac{W_{min}^2}{12X}$$

$$W_{min} = \sqrt{12MB_{min}X} = \sqrt{12 \times 8ft \times 8ft} = 27.71ft$$

Section 3 (15 points)

1) If the velocity of a liquid is 1.65 ft/s in a special pipe with an inside diameter of 12 in, what is the velocity in a 3-in-diameter jet exiting from a nozzle attached to the pipe? (5 points)
Solution:

$$A_1 v_1 = A_2 v_2;\ v_2 = v_1 \frac{A_1}{A_2} = v_1\left(\frac{D_1}{D_2}\right)^2 = \frac{1.65\ \text{ft}}{\text{s}}\left(\frac{12}{3}\right)^2 = \mathbf{26.4\ ft/s}$$

2) Water at 36 m above sea level have a velocity of 18 m/s and a pressure of 350 kN/m^2. Determine the pressure head, potential head, velocity head, and total head. $\gamma_{water} = 9.81 kN/\text{m}^3$.(5 points)
Solution:

Pressure Head:

$$\frac{p_1}{\gamma_{water}} = \frac{350 \times 10^3}{9.81} = 35.678m$$

Potential Head:

$$z_1 = 36m$$

Velocity Head:

$$\frac{\text{v}_1^2}{2g} = \frac{18^2}{2 \times 9.81} = 16.514m$$

Total Head:

$$\frac{p_1}{\gamma_{water}} + z_1 + \frac{\text{v}_1^2}{2g} = 35.678\text{m} + 36\text{m} + 16.514\text{m} = 88.192\ \text{m}$$

3) Calculate the maximum volume flow rate of fuel oil at 45 ℃ at which the flow will remain laminar in a steel pipe with an inside diameter of 0.0972 m. For the fuel oil, use density $\rho_{oil} = 895\text{kg/m}^3$, and dynamic viscosity of fuel oil at 45 ℃ η_{oil}=4.0 × 10^{-2} Pa.s. (5 points)
Solution:

To make the fuel oil flow as a laminar flow, $N_R \leq 2000$

$$N_R = \frac{\text{v}D\rho}{\eta} \leq 2000$$

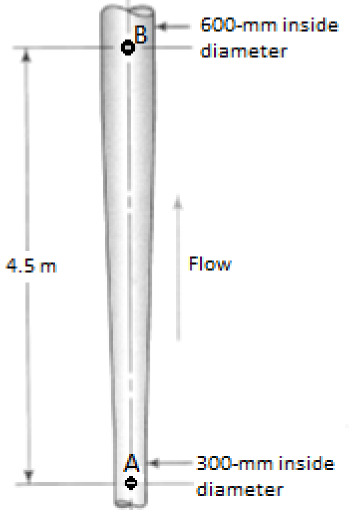
$$N_{R\_max} = 2000$$

$$\mathrm{v}_{max} = \frac{N_{R\_max}\eta}{D\rho} = \frac{2000 \times (4.0 \times 10^{-2})}{0.0972 \times 0.895 \times 1000} = 0.92 m/s$$

$$Q_{max} = \mathrm{v}_{max}A = \mathrm{v}_{max}\frac{\pi D^2}{4} = 0.92 \times \frac{\pi(0.0972)^2}{4} = 6.82 \times 10^{-3} m^3/s$$

Section 4 (40 points)

1) Water at 10℃ is flowing from Point A to Point B through the fabricated section shown in Fig. below at the rate of 0.37 m^3/s. If the pressure at A is 66.2 kPa, Assuming there are no energy losses in the system, calculate the pressure at B. The specific weight of water at 10℃, $\gamma_w = 9.81\ kN/m^3$. g=9.81m/s^2 (10 points)



Solution:

$$\frac{p_A}{\gamma_W} + z_A + \frac{\upsilon_A^2}{2g} = \frac{p_B}{\gamma_W} + z_B + \frac{\upsilon_B^2}{2g}; \upsilon_A = \frac{Q}{A_A} = \frac{0.37\ \mathrm{m^3/s}}{\pi(0.3\ \mathrm{m})^2/4} = \frac{5.23\ \mathrm{m}}{\mathrm{s}};$$

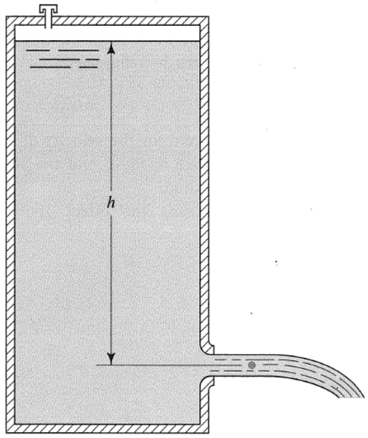
$$\upsilon_B = \upsilon_A\left(\frac{D_A}{D_B}\right)^2 = \frac{1.31\ \mathrm{m}}{\mathrm{s}}$$

$$p_B = p_A + \gamma_w\left[(z_A - z_B) + \frac{\upsilon_A^2 - \upsilon_B^2}{2g}\right]$$

$$= 66.2\ \mathrm{kPa} + \frac{9.81\ \mathrm{kN}}{\mathrm{m^3}}\left[-4.5 + \frac{(5.23^2 - 1.31^2)\mathrm{m^2/s^2}}{2(9.81\ \mathrm{m/s^2})}\right]$$

$$p_B = 66.2\ \mathrm{kPa} - 31.3\ \mathrm{kPa} = \mathbf{34.9\ kPa}$$

2) What depth (h) of fluid above the outlet nozzle is required to deliver 0.01 m^3/s of water from the tank shown below? The inside diameter of the nozzle is 0.0762 m. (10 points)



Solution:

$$Q_j = v_j A_j$$

$$v_j = \frac{Q_j}{A_j} = \frac{Q_j}{\frac{\pi D_j^2}{4}} = \frac{0.0126 m^3/s}{0.00456 m^2} = 2.763 m/s$$

$$h = \frac{v_j^2}{2g} = \frac{(2.763 m/s)^2}{2 \times 9.81 m/s^2} = 0.39\, m$$

3) A horizontal pipe (the inside diameter is constant) carries oil (its specific weight $\gamma_{oil} = 51.79\, lb/ft^3$). If two pressure gages along the pipe read 74.6 psig and 62.2 psig respectively, calculate the energy loss between the two gages. $1ft = 12in$. (10 points)
Solution:

$$\frac{p_1}{\gamma_o} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{p_2}{\gamma_o} + z_2 + \frac{v_2^2}{2g}$$

$$z_1 = z_2 \text{ and } \upsilon_1 = \upsilon_2$$

$$\frac{p_1}{\gamma_o} - h_L = \frac{p_2}{\gamma_o}$$

$$h_L = \frac{p_1}{\gamma_{oil}} - \frac{p_2}{\gamma_{oil}} = \frac{p_1 - p_2}{\gamma_{oil}} = \frac{74.6psi - 62.2psi}{51.79\, lb/ft^3}$$

$$= \frac{(74.6 - 62.2)psi}{51.79\, lb/ft^3} = \frac{(74.6 - 62.2) lb/in^2}{51.79\, lb/ft^3} = 34.5\, ft$$

4) Crude oil (its density $\rho_{oil} = 860\text{kg/m}^3$; its dynamic viscociy η_{oil}=1.7×10^{-2}Pa.s) is flowing vertically downward through 60 m of a steel pipe with an inside diameter of 0.0243m at a velocity of 0.64 m/s. Calculate the pressure drop over 60 m. g=9.81m/s^2. (10 points)
Solution:

$$\frac{p_1}{\gamma_o} + z_1 + \frac{\upsilon_1^2}{2g} - h_L = \frac{p_2}{\gamma_o} + z_2 + \frac{\upsilon_2^2}{2g}: \ \upsilon_1 = \upsilon_2$$

$$p_1 - p_2 = \gamma_o[z_2 - z_1 + h_L]$$

$$N_R = \frac{\text{v}D\rho}{\eta} = \frac{(0.64)(0.0243)(860)}{1.7 \times 10^{-2}} = 787$$

$N_R < 2000$ Laminar

$$f = \frac{64}{N_R} = 0.0813$$

$$h_L = f \ \frac{L}{D}\frac{\upsilon^2}{2g} = 0.0813 \times \frac{60}{0.0243} \times \frac{(0.64)^2}{2(9.81)} = 4.19 \text{ m}$$

$$p_1 - p_2 = \gamma_o[z_2 - z_1 + h_L]$$

$$p_1 - p_2 = (0.86)(9.82 \text{ kN/m}^3)[-60 \text{ m} + 4.19 \text{ m}] = -471 \text{ kN/m}^2 = \mathbf{-471 \ kPa}$$