

5-6 ■ GENERAL ENERGY EQUATION

One of the most fundamental laws in nature is the **first law of thermodynamics**, also known as the **conservation of energy principle**, which provides a sound basis for studying the relationships among the various forms of energy and energy interactions. It states that *energy can be neither created nor destroyed during a process; it can only change forms*. Therefore, every bit of energy must be accounted for during a process.

A rock falling off a cliff, for example, picks up speed as a result of its potential energy being converted to kinetic energy (Fig. 5-43). Experimental data show that the decrease in potential energy equals the increase in kinetic energy when the air resistance is negligible, thus confirming the conservation of energy principle. The conservation of energy principle also forms the backbone of the diet industry: a person who has a greater energy input (food) than energy output (exercise) will gain weight (store energy in the form of fat), and a person who has a smaller energy input than output will lose weight. The change in the energy content of a system is equal to the difference between the energy input and the energy output, and the conservation of energy principle for any system can be expressed simply as $E_{in} - E_{out} = \Delta E$.

The transfer of any quantity (such as mass, momentum, and energy) is recognized *at the boundary* as the quantity *crosses the boundary*. A quantity is said to *enter* a system if it crosses the boundary from the outside to the inside, and to *exit* the system if it moves in the reverse direction. A quantity that moves from one location to another within a system is not considered as a transferred quantity in an analysis since it does not enter or exit the system. Therefore, it is important to specify the system and thus clearly identify its boundaries before an engineering analysis is performed.

The energy content of a fixed quantity of mass (a closed system) can be changed by two mechanisms: *heat transfer* Q and *work transfer* W . Then the conservation of energy for a fixed quantity of mass can be expressed in rate form as (Fig. 5-44)

$$\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{dE_{sys}}{dt} \quad \text{or} \quad \dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{d}{dt} \int_{sys} \rho e\ dV \quad (5-49)$$

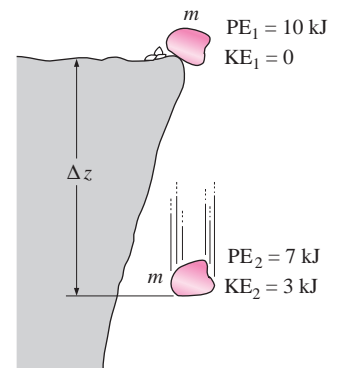


FIGURE 5-43

Energy cannot be created or destroyed during a process; it can only change forms.

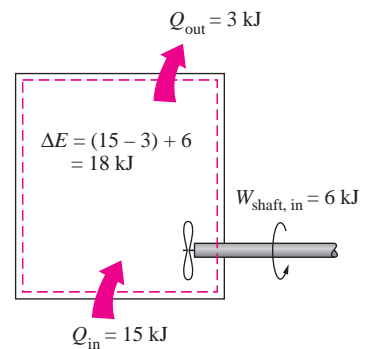


FIGURE 5-44

The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

where $\dot{Q}_{\text{net in}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$ is the net rate of heat transfer to the system (negative, if from the system), $\dot{W}_{\text{net in}} = \dot{W}_{\text{in}} - \dot{W}_{\text{out}}$ is the net power input to the system in all forms (negative, if power output) and dE_{sys}/dt is the rate of change of the total energy content of the system. The overdot stands for time rate. For simple compressible systems, total energy consists of internal, kinetic, and potential energies, and it is expressed on a unit-mass basis as (see Chap. 2)

$$e = u + ke + pe = u + \frac{V^2}{2} + gz \quad (5-50)$$

Note that total energy is a property, and its value does not change unless the state of the system changes.

Energy Transfer by Heat, Q

In daily life, we frequently refer to the sensible and latent forms of internal energy as *heat*, and talk about the heat content of bodies. Scientifically the more correct name for these forms of energy is *thermal energy*. For single-phase substances, a change in the thermal energy of a given mass results in a change in temperature, and thus temperature is a good representative of thermal energy. Thermal energy tends to move naturally in the direction of decreasing temperature, and the transfer of thermal energy from one system to another as a result of a temperature difference is called **heat transfer**. Therefore, an energy interaction is heat transfer only if it takes place because of a temperature difference. The warming up of a canned drink in a warmer room, for example, is due to heat transfer (Fig. 5–45). The time rate of heat transfer is called **heat transfer rate** and is denoted by \dot{Q} .

The direction of heat transfer is always from the higher-temperature body to the lower-temperature one. Once temperature equality is established, heat transfer stops. There cannot be any heat transfer between two systems (or a system and its surroundings) that are at the same temperature.

A process during which there is no heat transfer is called an **adiabatic process**. There are two ways a process can be adiabatic: Either the system is well insulated so that only a negligible amount of heat can pass through the system boundary, or both the system and the surroundings are at the same temperature and therefore there is no driving force (temperature difference) for heat transfer. An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work transfer.

Energy Transfer by Work, W

An energy interaction is **work** if it is associated with a force acting through a distance. A rising piston, a rotating shaft, and an electric wire crossing the system boundary are all associated with work interactions. The time rate of doing work is called **power** and is denoted by \dot{W} . Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.

Work-consuming devices transfer energy to the fluid, and thus increase the energy of the fluid. A fan in a room, for example, mobilizes the air and increases its kinetic energy. The electric energy a fan consumes is first converted to mechanical energy by its motor that forces the shaft of the blades

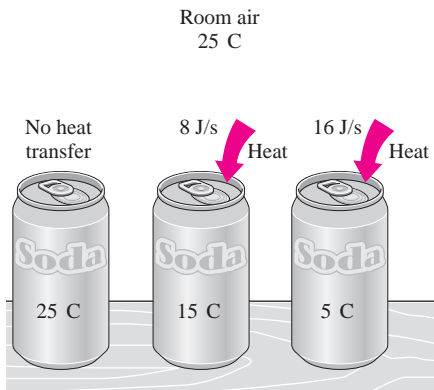


FIGURE 5–45

Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.

to rotate. This mechanical energy is then transferred to the air, as evidenced by the increase in air velocity. This energy transfer to air has nothing to do with a temperature difference, so it cannot be heat transfer. Therefore, it must be work. Air discharged by the fan eventually comes to a stop and thus loses its mechanical energy as a result of friction between air particles of different velocities. But this is not a “loss” in the real sense; it is simply the conversion of mechanical energy to an equivalent amount of thermal energy (which is of limited value, and thus the term *loss*) in accordance with the conservation of energy principle. If a fan runs a long time in a sealed room, we can sense the buildup of this thermal energy by a rise in air temperature.

A system may involve numerous forms of work, and the total work can be expressed as

$$W_{\text{total}} = W_{\text{shaft}} + W_{\text{pressure}} + W_{\text{viscous}} + W_{\text{other}} \quad (5-51)$$

where W_{shaft} is the work transmitted by a rotating shaft, W_{pressure} is the work done by the pressure forces on the control surface, W_{viscous} is the work done by the normal and shear components of viscous forces on the control surface, and W_{other} is the work done by other forces such as electric, magnetic, and surface tension, which are insignificant for simple compressible systems and are not considered in this text. We do not consider W_{viscous} either since it is usually small relative to other terms in control volume analysis. But it should be kept in mind that the work done by shear forces as the blades shear through the fluid may need to be considered in a refined analysis of turbomachinery.

Shaft Work

Many flow systems involve a machine such as a pump, a turbine, a fan, or a compressor whose shaft protrudes through the control surface, and the work transfer associated with all such devices is simply referred to as *shaft work* W_{shaft} . The power transmitted via a rotating shaft is proportional to the shaft torque T_{shaft} and is expressed as

$$\dot{W}_{\text{shaft}} = \omega T_{\text{shaft}} = 2\pi \dot{n} T_{\text{shaft}} \quad (5-52)$$

where ω is the angular speed of the shaft in rad/s and \dot{n} is defined as the number of revolutions of the shaft per unit time, often expressed in rev/min or rpm.

Work Done by Pressure Forces

Consider a gas being compressed in the piston-cylinder device shown in Fig. 5-46a. When the piston moves down a differential distance ds under the influence of the pressure force PA , where A is the cross-sectional area of the piston, the boundary work done *on* the system is $\delta W_{\text{boundary}} = PA ds$. Dividing both sides of this relation by the differential time interval dt gives the time rate of boundary work (i.e., *power*),

$$\delta \dot{W}_{\text{pressure}} = \delta \dot{W}_{\text{boundary}} = PAV_{\text{piston}}$$

where $V_{\text{piston}} = ds/dt$ is the piston velocity, which is the velocity of the moving boundary at the piston face.

Now consider a material chunk of fluid (a system) of arbitrary shape, which moves with the flow and is free to deform under the influence of pressure, as shown in Fig. 5-46b. Pressure always acts inward and normal

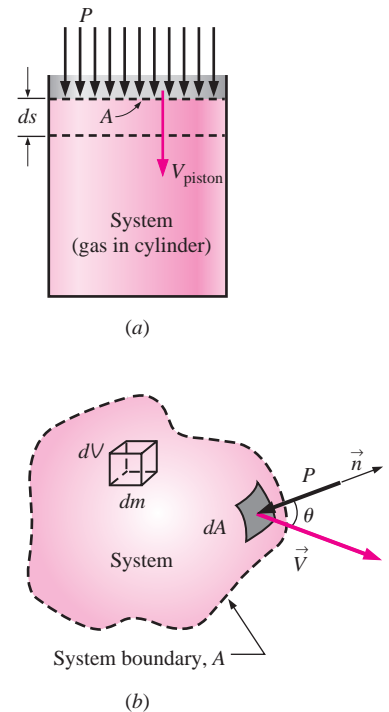


FIGURE 5-46

The pressure force acting on (a) the moving boundary of a system in a piston-cylinder device, and (b) the differential surface area of a system of arbitrary shape.

to the surface, and the pressure force acting on a differential area dA is $P dA$. Again noting that work is force times distance and distance traveled per unit time is velocity, the time rate at which work is done by pressure forces on this differential part of the system is

$$\delta \dot{W}_{\text{pressure}} = -P dA V_n = -P dA(\vec{V} \cdot \vec{n}) \quad (5-53)$$

since the normal component of velocity through the differential area dA is $V_n = V \cos \theta = \vec{V} \cdot \vec{n}$. Note that \vec{n} is the outer normal of dA , and thus the quantity $\vec{V} \cdot \vec{n}$ is positive for expansion and negative for compression. The negative sign in Eq. 5-53 ensures that work done by pressure forces is positive when it is done *on* the system, and negative when it is done *by* the system, which agrees with our sign convention. The total rate of work done by pressure forces is obtained by integrating $\delta \dot{W}_{\text{pressure}}$ over the entire surface A ,

$$\dot{W}_{\text{pressure, net in}} = - \int_A P(\vec{V} \cdot \vec{n}) dA = - \int_A \frac{P}{\rho} \rho(\vec{V} \cdot \vec{n}) dA \quad (5-54)$$

In light of these discussions, the net power transfer can be expressed as

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \dot{W}_{\text{shaft, net in}} - \int_A P(\vec{V} \cdot \vec{n}) dA \quad (5-55)$$

Then the rate form of the conservation of energy relation for a closed system becomes

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{dE_{\text{sys}}}{dt} \quad (5-56)$$

To obtain a relation for the conservation of energy for a *control volume*, we apply the Reynolds transport theorem by replacing B with total energy E , and b with total energy per unit mass e , which is $e = u + ke + pe = u + V^2/2 + gz$ (Fig. 5-47). This yields

$$\frac{dE_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{CV}} e\rho dV + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n}) dA \quad (5-57)$$

Substituting the left-hand side of Eq. 5-56 into Eq. 5-57, the general form of the energy equation that applies to fixed, moving, or deforming control volumes becomes

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{d}{dt} \int_{\text{CV}} e\rho dV + \int_{\text{CS}} e\rho(\vec{V}_r \cdot \vec{n}) dA \quad (5-58)$$

which can be stated as

$$\left(\begin{array}{l} \text{The net rate of energy} \\ \text{transfer into a CV by} \\ \text{heat and work transfer} \end{array} \right) = \left(\begin{array}{l} \text{The time rate of} \\ \text{change of the energy} \\ \text{content of the CV} \end{array} \right) + \left(\begin{array}{l} \text{The net flow rate of} \\ \text{energy out of the control} \\ \text{surface by mass flow} \end{array} \right)$$

Here $\vec{V}_r = \vec{V} - \vec{V}_{\text{CS}}$ is the fluid velocity relative to the control surface, and the product $\rho(\vec{V}_r \cdot \vec{n}) dA$ represents the mass flow rate through area element dA into or out of the control volume. Again noting that \vec{n} is the outer normal of dA , the quantity $\vec{V}_r \cdot \vec{n}$ and thus mass flow is positive for outflow and negative for inflow.

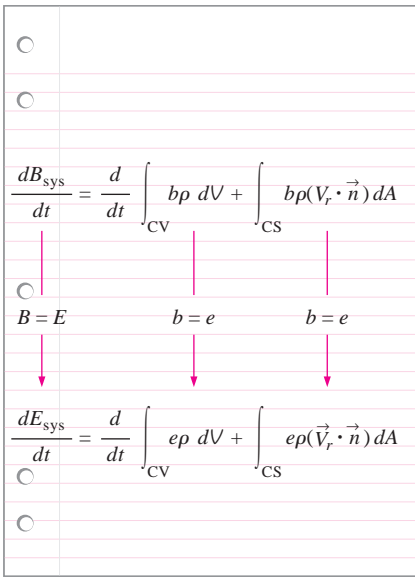


FIGURE 5-47

The conservation of energy equation is obtained by replacing B in the Reynolds transport theorem by energy E and b by e .

Substituting the surface integral for the rate of pressure work from Eq. 5–54 into Eq. 5–58 and combining it with the surface integral on the right give

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt} \int_{\text{CV}} e \rho \, dV + \int_{\text{CS}} \left(\frac{P}{\rho} + e \right) \rho (\vec{V}_r \cdot \vec{n}) \, dA \quad (5-59)$$

This is a very convenient form for the energy equation since pressure work is now combined with the energy of the fluid crossing the control surface and we no longer have to deal with pressure work.

The term $P/\rho = P\nu = w_{\text{flow}}$ is the **flow work**, which is the work associated with pushing a fluid into or out of a control volume per unit mass. Note that the fluid velocity at a solid surface is equal to the velocity of the solid surface because of the no-slip condition and is zero for nonmoving surfaces. As a result, the pressure work along the portions of the control surface that coincide with nonmoving solid surfaces is zero. Therefore, pressure work for fixed control volumes can exist only along the imaginary part of the control surface where the fluid enters and leaves the control volume, i.e., inlets and outlets.

For a fixed control volume (no motion or deformation of control volume), $\vec{V}_r = \vec{V}$ and the energy equation Eq. 5–59 becomes

Fixed CV:
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt} \int_{\text{CV}} e \rho \, dV + \int_{\text{CS}} \left(\frac{P}{\rho} + e \right) \rho (\vec{V} \cdot \vec{n}) \, dA \quad (5-60)$$

This equation is not in a convenient form for solving practical engineering problems because of the integrals, and thus it is desirable to rewrite it in terms of average velocities and mass flow rates through inlets and outlets. If $P/\rho + e$ is nearly uniform across an inlet or outlet, we can simply take it outside the integral. Noting that $\dot{m} = \int_{A_c} \rho (\vec{V} \cdot \vec{n}) \, dA_c$ is the mass flow rate across an inlet or outlet, the rate of inflow or outflow of energy through the inlet or outlet can be approximated as $\dot{m}(P/\rho + e)$. Then the energy equation becomes (Fig. 5–48)

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt} \int_{\text{CV}} e \rho \, dV + \sum_{\text{out}} \dot{m} \left(\frac{P}{\rho} + e \right) - \sum_{\text{in}} \dot{m} \left(\frac{P}{\rho} + e \right) \quad (5-61)$$

where $e = u + V^2/2 + gz$ (Eq. 5–50) is the total energy per unit mass for both the control volume and flow streams. Then,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt} \int_{\text{CV}} e \rho \, dV + \sum_{\text{out}} \dot{m} \left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz \right) \quad (5-62)$$

or

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt} \int_{\text{CV}} e \rho \, dV + \sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) \quad (5-63)$$

where we used the definition of enthalpy $h = u + P\nu = u + P/\rho$. The last two equations are fairly general expressions of conservation of energy, but their use is still limited to fixed control volumes, uniform flow at inlets and

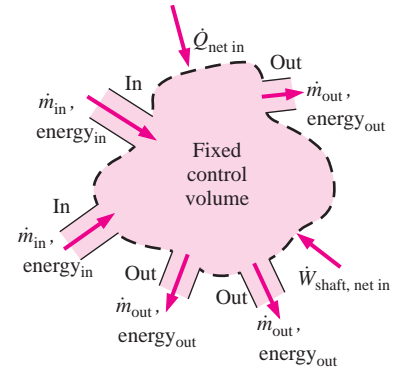


FIGURE 5–48

In a typical engineering problem, the control volume may contain many inlets and outlets; energy flows in at each inlet, and energy flows out at each outlet. Energy also enters the control volume through net heat transfer and net shaft work.

outlets, and negligible work due to viscous forces and other effects. Also, the subscript “net in” stands for “net input,” and thus any heat or work transfer is positive if *to* the system and negative if *from* the system.

5-7 ■ ENERGY ANALYSIS OF STEADY FLOWS

For steady flows, the time rate of change of the energy content of the control volume is zero, and Eq. 5-63 simplifies to

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) - \sum_{\text{in}} \dot{m} \left(h + \frac{V^2}{2} + gz \right) \quad (5-64)$$

It states that *the net rate of energy transfer to a control volume by heat and work transfers during steady flow is equal to the difference between the rates of outgoing and incoming energy flows with mass.*

Many practical problems involve just one inlet and one outlet (Fig. 5-49). The mass flow rate for such **single-stream devices** remains constant, and Eq. 5-64 reduces to

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m} \left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right) \quad (5-65)$$

where subscripts 1 and 2 stand for inlet and outlet, respectively. The steady-flow energy equation on a unit-mass basis is obtained by dividing Eq. 5-65 by the mass flow rate \dot{m} ,

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad (5-66)$$

where $q_{\text{net in}} = \dot{Q}_{\text{net in}}/\dot{m}$ is the net heat transfer to the fluid per unit mass and $w_{\text{shaft, net in}} = \dot{W}_{\text{shaft, net in}}/\dot{m}$ is the net shaft work input to the fluid per unit mass. Using the definition of enthalpy $h = u + P/\rho$ and rearranging, the steady-flow energy equation can also be expressed as

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}}) \quad (5-67)$$

where u is the *internal energy*, P/ρ is the *flow energy*, $V^2/2$ is the *kinetic energy*, and gz is the *potential energy* of the fluid, all per unit mass. These relations are valid for both compressible and incompressible flows.

The left side of Eq. 5-67 represents the mechanical energy input, while the first three terms on the right side represent the mechanical energy output. If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved, and the term in parentheses ($u_2 - u_1 - q_{\text{net in}}$) must equal zero. That is,

$$\text{Ideal flow (no mechanical energy loss):} \quad q_{\text{net in}} = u_2 - u_1 \quad (5-68)$$

Any increase in $u_2 - u_1$ above $q_{\text{net in}}$ is due to the irreversible conversion of mechanical energy to thermal energy, and thus $u_2 - u_1 - q_{\text{net in}}$ represents the mechanical energy loss (Fig. 5-50). That is,

$$\text{Mechanical energy loss:} \quad e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}} \quad (5-69)$$

For single-phase fluids (a gas or a liquid), we have $u_2 - u_1 = c_v(T_2 - T_1)$ where c_v is the constant-volume specific heat.

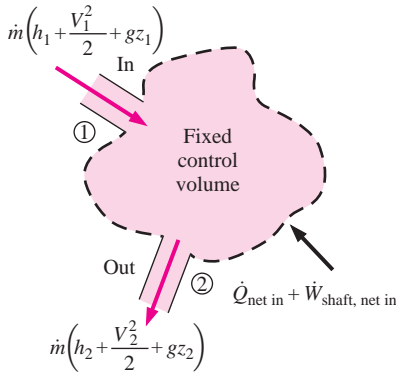


FIGURE 5-49

A control volume with only one inlet and one outlet and energy interactions.

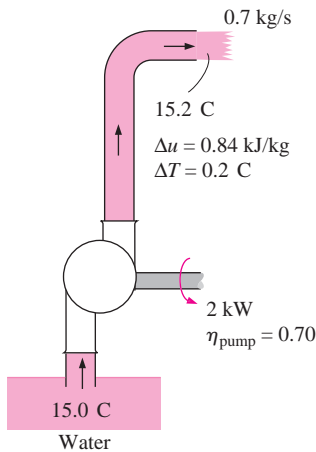


FIGURE 5-50

The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.

The steady-flow energy equation on a unit-mass basis can be written conveniently as a **mechanical energy** balance as

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}} \quad (5-70)$$

or

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}} \quad (5-71)$$

Noting that $w_{\text{shaft, net in}} = w_{\text{shaft, in}} - w_{\text{shaft, out}} = w_{\text{pump}} - w_{\text{turbine}}$, the mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}} \quad (5-72)$$

where w_{pump} is the mechanical work input (due to the presence of a pump, fan, compressor, etc.) and w_{turbine} is the mechanical work output. When the flow is incompressible, either absolute or gage pressure can be used for P since P_{atm}/ρ would appear on both sides and would cancel out.

Multiplying Eq. 5-72 by the mass flow rate \dot{m} gives

$$\dot{m} \left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 \right) + \dot{W}_{\text{pump}} = \dot{m} \left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 \right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \quad (5-73)$$

where \dot{W}_{pump} is the shaft power input through the pump's shaft, \dot{W}_{turbine} is the shaft power output through the turbine's shaft, and $\dot{E}_{\text{mech, loss}}$ is the *total* mechanical power loss, which consists of pump and turbine losses as well as the frictional losses in the piping network. That is,

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

By convention, irreversible pump and turbine losses are treated separately from irreversible losses due to other components of the piping system. Thus the energy equation can be expressed in its most common form in terms of heads as (Fig. 5-51).

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L \quad (5-74)$$

where $h_{\text{pump, } u} = \frac{w_{\text{pump, } u}}{g} = \frac{\dot{W}_{\text{pump, } u}}{\dot{m}g} = \frac{\eta_{\text{pump}} \dot{W}_{\text{pump}}}{\dot{m}g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{\text{pump, } u}$ is *less* than $\dot{W}_{\text{pump}}/\dot{m}g$ by the factor η_{pump} . Similarly,

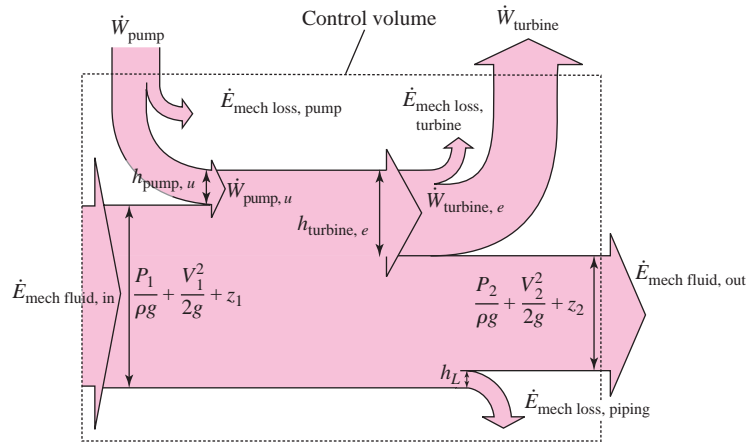
$h_{\text{turbine, } e} = \frac{w_{\text{turbine, } e}}{g} = \frac{\dot{W}_{\text{turbine, } e}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}} \dot{m}g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{\text{turbine, } e}$ is *greater* than $\dot{W}_{\text{turbine}}/\dot{m}g$ by the factor η_{turbine} . Finally,

$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$ is the *irreversible head loss* between

1 and 2 due to all components of the piping system other than the pump or turbine. Note that the head loss h_L represents the frictional losses associated with fluid flow in piping, and it does not include the losses that occur within the pump or turbine due to the inefficiencies of these devices—these losses are taken into account by η_{pump} and η_{turbine} . Equation 5-74 is illustrated schematically in Fig. 5-51.

FIGURE 5-51

Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, i.e., *head*, corresponding to each term of Eq. 5-74.



The *pump head* is zero if the piping system does not involve a pump, a fan, or a compressor, and the *turbine head* is zero if the system does not involve a turbine. Also, the *head loss* h_L can sometimes be ignored when the frictional losses in the piping system are negligibly small compared to the other terms in Eq. 5-74.

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{\text{mech loss, piping}}/g \cong 0$, as shown later in Example 5-11. Also, $h_{\text{pump, u}} = h_{\text{turbine, e}} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 5-74 reduces to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \text{or} \quad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \quad (5-75)$$

which is the **Bernoulli equation** derived earlier using Newton's second law of motion.