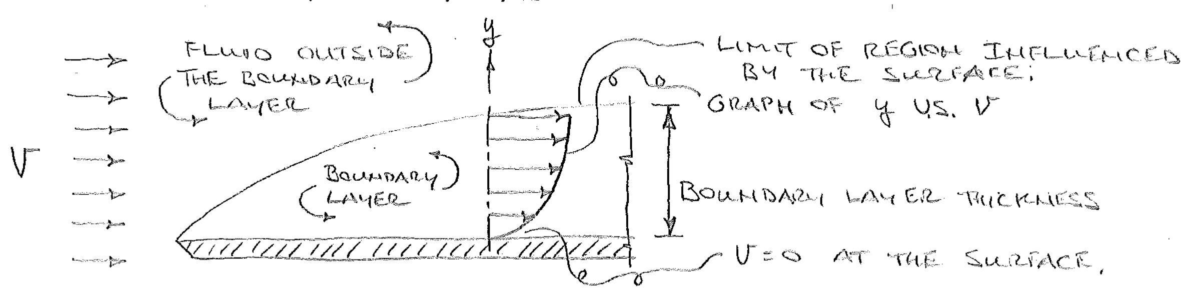
Heat Transfer

Calculation of the Convective Heat Transfer Coefficient

Fluid Velocity Boundary Layer

Imagine that a fluid (gas or liquid) is flowing along. If you look around inside the fluid you will find places where the fluid comes in contact with solid objects. The solid objects might be the walls of a room, your skin, the inside surface of a coffee cup – really any solid at all. If you were to take a really close look at the place where the solid and fluid meet you will find that the fluid atoms are stuck to the solid. The fluid is so stuck that right at the solid surface the fluid is not moving. Just a few atoms away from the surface the fluid atoms are moving. They slide past the fluid atoms stuck to the wall but are slowed by the attractive forces that exist between themselves and the stationary atoms stuck to the wall. Those slower fluid atoms act to slow the fluid atoms next to them and so on. At some small distance away from the solid surface you will find that the fluid is moving along at the bulk fluid velocity, as though the solid surface is not there at all.

The layer of fluid atoms slowed, or as they say "influenced", by the presence of the solid wall is called the Velocity Boundary Layer.



Within the Velocity Boundary Layer there are two competing forces that tend to control how the fluid moves across the solid surface. Inertial forces arise from the fluids velocity and tend to amplify any disturbances in the flow. On the other hand, Viscous forces arise from the attractive forces between fluid atoms and tend to dampen the attempt by the inertial forces to make the flow unsteady.

At low velocities the inertial forces are small and the viscous forces are relatively large so the fluid flows smoothly in a well organized pattern called *Laminar Flow*. At higher velocities the inertial forces are large and tend to dominate the relatively smaller viscous forces so the flow has a tendency to move in a highly disorganized way known as *Turbulent Flow*.

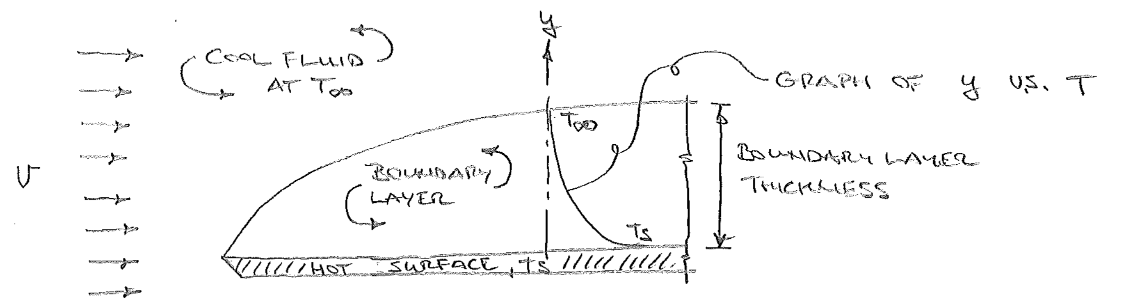
It is possible to numerically predict whether the flow will be laminar or turbulent by calculating the *Reynolds Number*. The Reynolds Number is simply a calculated ratio of the Inertial Forces over the Viscous Forces. The equation looks like this.

$$\mathrm{Re} = \frac{VL\rho}{\mu} \qquad \textit{also} \qquad \mathrm{Re} = \frac{VL}{\nu}$$

Where:
- V = Flow Velocity outside the Boundary Layer, m/s
- L = Dimensional Parameter, m
- ρ = Fluid density, kg/m^3
- μ = Dynamic viscosity, kg/m-s
- ν = Kinematic viscosity, m^2/s $= \mu/\rho$

You will notice that, in a way, Re is simply a dimensionless form of velocity. The size of the number that emerges from the equation is an indicator of the type of flow that will exist. For flow moving inside a pipe, for example, laminar flow occurs when Re is less than 2000 and turbulent flow occurs when Re is greater than 4000. Between 2000 and 4000 the flow can be either laminar or turbulent – its unpredictable.

Fluid Thermal Boundary Layer
When a cool fluid moves across a warm surface it warms up. The fluid close to the warm surface becomes hotter than the fluid farther away from the surface. As a result of the observed fluid temperature distribution near the surface it is said that the fluid has a Thermal Boundary Layer. Like the Velocity Boundary Layer, the Thermal Boundary Layer is the layer of fluid close to a surface that is "influenced" by the surface. Its shape and thickness are usually different than the Velocity Boundary Layer but the way heat is transferred through the Thermal Boundary Layer is definitely affected by the nature of flow near the surface (laminar or turbulent).



Convective Heat Transfer
Recall that we have discussed the calculation of two major types of heat transfer: Conductive and Convective.

One can determine the rate of heat transfer in each of these forms as follows:

Conductive Heat Transfer Equation: $\dot{Q}_{COND} = \frac{kA}{t}(T_{HOT} - T_{COLD})$

Convective Heat Transfer Equation: $\dot{Q}_{CONV} = hA(T_S - T_\infty)$

To find "k", the Conductive Heat Transfer Coefficient, you just have to look up its value in the tables in the appendix of this document.

To find "h", the Convective Heat Transfer Coefficient, is a bit more work. The next few pages hold a step-by-step method of calculating "h". As part of the calculation process you will need to look up certain fluid properties in the fluid tables in the appendix this document.

(I'm sure you are aware that when looking up properties you simply need to determine the material through which the heat is passing, find that material in the tables and look in the appropriate column for the property. A trick to make sure you've found the right property is to check your units. If the units of the property seem to match the units required in an equation you probably are looking in the correct column.)

Calculating "h" – The Convective Heat Transfer Coefficient
Lots of experiments have been done relating "h" with various fluid properties, different flow types and a wide variety of surface shapes. Several important relationships have immerged from this work.

Nusselt Number: $$Nu = \frac{hL}{k}$$

Where: h = Convective heat transfer coefficient, W/m^2-C
L = Dimensional Parameter, m
k = Thermal Conductivity, W/m-C

Your goal is to find "h" ... so ... just solve for it. If you have Nu, L and k, you will have "h".

$$h = Nu\frac{k}{L}$$

The trick is to find Nu, the Nusselt Number. Experiments have shown that Nu is related to two or three fluid properties: the Reynolds Number, Re (that we just looked at), the Prandtl Number, Pr and (sometimes) the Grashof Number, Gr.

Prandtl Number: $$\Pr = \frac{\mu C_P}{k}$$

Where: C_P = Specific Heat of the fluid, J/kg-C

Often the Prandtl number is given directly in the tables of properties so you usually don't have to calculate it. You may have already noticed that it's another dimensionless number.

Grashof Number: $$Gr = \frac{g\beta(T_{SURFACE} - T_\infty)L^3\rho^2}{\mu^2}$$

Where: g = gravitational acceleration constant = 9.81 m/s^2
β = Isobaric Compressibility, 1/C (See note below.)

Note: See the section entitled "A Few Notes About Fluids and Heat" for more information on how to determine Cp, β and ρ.

Steps-by-Step Method finding "h"
1. Look at the shape and size of the convective heat transfer situation. Based on the different flow types and the illustrations presented in Table C-8 below, determine the geometric parameter, L. (Notice in some situation the geometric parameter is shown using the variable D for diameter. D and L are interchangeable.)
2. Determine if a fan or pump forces the flow to move (Forced Convection: Flow situations A, B and C on Table C-8 below) or if the flow moves naturally as a result of buoyancy forces (Natural convection: Flow situation D on Table C-8 below).
3. Determine the type of fluid being used to convect away the heat. Look-up the thermo-physical properties of that fluid in the tables provided.

4. Calculate the Reynolds Number, Re and the Prandtl Number (if its not given in the tables).
   If the flow is of the Natural Convection Type you will also have to calculate the Grashof Number, Gr.
5. Calculate Nu.
6. Calculate h.
7. Do what ever the problem asks you to do.

A few Notes About Fluids and Heat

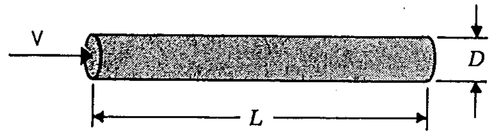
Thing 1: The tables give very complete property information for liquids. Just look up the properties – no big deal.

Thing 2: If you are dealing with a gas, like air, you will find the tables lacking when it comes to finding β. For most simple (ideal) gasses finding it is easy.

$\beta = \frac{1}{T}$ (For gasses only!) Where: T = Absolute temperature, K
OF THE FLUID

Table C•8 Convection heat transfer correlations

A. Flow in circular tubes



$$\mathrm{Re} \equiv \frac{VD\rho}{\mu}$$

$$\mathrm{Nu} \equiv \frac{hD}{k}$$

$$h \equiv \frac{q''}{\Delta T}$$

$$\Delta T = T_{\text{wall}} - T_{\text{mean}}$$

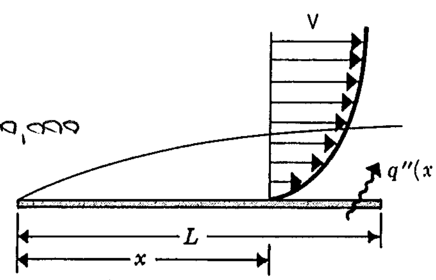
1. *Laminar flow,* $\mathrm{Re} < 2000$

   Fully developed flow $\dfrac{L/D}{\mathrm{RePr}} > 0.05$

   $\mathrm{Nu} = 4.364$ *uniform wall heat flux*
   $\mathrm{Nu} = 3.66$ *uniform wall temperature*

2. *Turbulent flow,* $\mathrm{Re} > 2000$
   $\mathrm{Pr} < 0.1$ *(liquid metals)* $10^2 < (\mathrm{RePr}) < 10^4$
   $\mathrm{Nu} = 4.82 + 0.0185\,(\mathrm{RePr})^{0.827}$ *uniform wall heat flux*
   $0.5 < \mathrm{Pr} < 1.0$ *(gases)*
   $\mathrm{Nu} = 0.022\,\mathrm{Pr}^{0.6}\mathrm{Re}^{0.8}$
   $1.0 < \mathrm{Pr} < 20$ *water and light oils*
   $\mathrm{Nu} = 0.0155\,\mathrm{Pr}^{0.5}\mathrm{Re}^{0.83}$
   $\mathrm{Pr} > 20$
   $\mathrm{Nu} = 0.0118\,\mathrm{Pr}^{0.3}\mathrm{Re}^{0.9}$

B. Boundary layer on a flat plate NOTE: $Re_{CR} = 500{,}000$



$$q''_{\text{av}} \equiv \frac{1}{L}\int_0^L q''(x)\,dX \qquad \Delta T \equiv T_{\text{wall}} - T_{\text{flow}}$$

$$h_x \equiv \frac{q''}{\Delta T} \qquad h_L \equiv \frac{q''_{\text{av}}}{\Delta T}$$

$$\mathrm{Nu}_x \equiv \frac{h_x x}{k} \qquad \mathrm{Nu}_L \equiv \frac{h_L L}{k} \qquad \mathrm{Re}_L = \frac{V\rho L}{\mu} \qquad \mathrm{Re}_x = \frac{V\rho x}{\mu}$$

1. *Laminar flow,* $\mathrm{Re}_L <$ 500,000
   $0.5 < \mathrm{Pr} < 15$

   $$\left.\begin{aligned}\mathrm{Nu}_x &= 0.332\,\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}\\ \mathrm{Nu}_L &= 0.664\,\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}\end{aligned}\right\}\ \textit{uniform wall temperature}$$

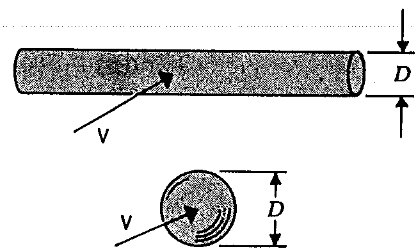
   $$\left.\begin{aligned}\mathrm{Nu}_x &= 0.453\,\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}\\ \mathrm{Nu}_L &= 0.906\,\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}\end{aligned}\right\}\ \textit{uniform wall heat flux}$$

2. *Turbulent flow,* $\mathrm{Re}_L >$ 500,000 AND $Re_L \leq 10{,}000{,}000$
   $0.5 < \mathrm{Pr} < 60$
   $\mathrm{Nu}_x = 0.0295\,\mathrm{Pr}^{1/3}\mathrm{Re}^{0.8}$

   $Nu_L = 0.037\,Pr^{1/3}\,Re_L^{0.8}$

Table C·8 (*continued*)

C. Single circular cylinder or sphere in cross flow



$$\mathrm{Re} \equiv \frac{VD\rho}{\mu} \qquad \mathrm{Nu} \equiv \frac{hD}{k}$$

$$h \equiv \frac{q''_{av}}{\Delta T} \qquad \Delta T = T_{wall} - T_{fluid}$$

1. Cylinder
$\mathrm{Nu} = C(\mathrm{Re})^n$ NOTE: $\mathrm{Re}_{CR} = 200{,}000$

Re	C, gases	C, liquids	n
0.4 - 4	0.891	0.989 $\mathrm{Pr}^{1/3}$	0.330
4–40	0.821	0.911 $\mathrm{Pr}^{1/3}$	0.385
40–4000	0.615	0.683 $\mathrm{Pr}^{1/3}$	0.466
4000–40,000	0.174	0.193 $\mathrm{Pr}^{1/3}$	0.618
40,000–400,000	0.0239	0.0266 $\mathrm{Pr}^{1/3}$	0.805

(USE FOR GAS... IN A PINCH)

2. Sphere NOTE: $\mathrm{Re}_{CR} = 200{,}000$
$\mathrm{Nu} = 0.37\,\mathrm{Re}^{0.6}$
$17 < \mathrm{Re} < 70{,}000$, gases
$\mathrm{Nu} = (1.2 + 0.53\,\mathrm{Re}_D^{0.54})\,\mathrm{Pr}^{0.3}$
$1 < \mathrm{Re} < 200{,}000$, $\mathrm{Pr} > 3$

D. Natural convection from a horizontal cylinder (Nu, Gr *based on diameter D*)
$\mathrm{Nu} = C(\mathrm{GrPr})^n$

GrPr	C	n
10^3–10^9	0.53	$\frac{1}{4}$
10^9–10^{12}	0.13	$\frac{1}{3}$

E. Natural convection from vertical surfaces (Nu, Gr *based on height L*)
$\mathrm{Nu} = C(\mathrm{GrPr})^n$

GrPr	C	n
10^5–10^9	0.555	0.25
$> 10^9$	0.021	0.4

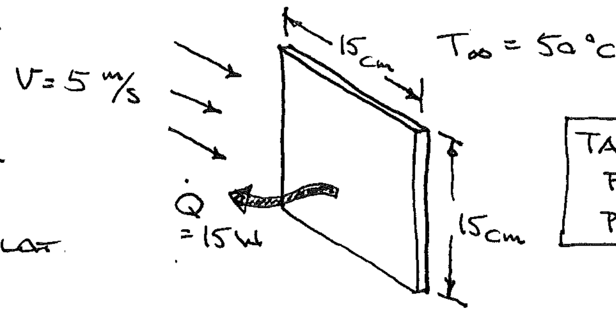
Compiled from W. M. Kays, *Convective Heat and Mass Transfer*, McGraw-Hill Book Company, New York, 1966; and W. H. McAdams, *Heat Transmission*, 3d ed., McGraw-Hill Book Company, New York, 1954.

EXAMPLE

A VERTICALLY ORIENTED 15cm × 15cm CIRCUIT BOARD UNIFORMLY DISSIPATES 15 WATTS OF HEAT FROM ITS COMPONENT SIDE ONLY. IT IS COOLED BY AIR AT 50°C

FOR THE 3 SITUATIONS OUTLINED BELOW FIND THE SURFACE TEMPERATURE OF THE BOARD.

a) FORCED CONVECTION WITH AN AIR VELOCITY OF 5 m/s.
b) FORCED CONVECTION WITH AN AIR VELOCITY OF 5 m/s AND ASSUMING A TURBULENT FLOW AS THE COMPONENTS ACT AS TURBULATORS.
c) NATURAL CONVECTION



TAKE $T_f = 60°C$ FOR THIS PROBLEM.

a & b

STEP ①: SHAPE, SIZE & L
L = 0.15 m
GEOMETRY Ⓑ FLAT PLATE, C-8.

STEP ②: FORCED CONVECTION

STEP ③: FLUID PROPERTIES → 60°C = 333°K ≈ 330K.
FROM TABLE A-19 FOR AIR
$\rho = 1.079$ kg/m³, $k = 0.0283$ W/m·°C, 1.99×10^{-5} kg/m·s
$\nu = 1.86 \times 10^{-5}$ m²/s, $Pr = 0.708$

STEP ④: $Re = \frac{VL}{\nu} = \frac{(5 \text{ m/s})(0.15 \text{ m})}{1.86 \times 10^{-5} \text{ m}^2/\text{s}} = 4.03 \times 10^4 = 40{,}300$

a) $Re < 500{,}000$ ∴ LAMINAR FLOW.

b) TURBULENT FLOW DUE TO COMPONENTS.

STEP ⑤: $Nu = ?$

a) $Nu = 0.906 \, Re^{1/2} \, Pr^{1/3}$ (UNIFORM WALL HEAT FLUX).
$= 0.906 (40{,}300)^{1/2} (0.708)^{1/3}$
$= 169$

b) $Nu = 0.037 \, Re^{0.8} \, Pr^{1/3}$ (TURBULENT).
$= 0.037 (40{,}300)^{0.8} (0.708)^{1/3}$
$= 167$

STEP ⑥ : $h = ?$

a) $h = Nu \frac{k}{L} = (169)\frac{(0.0283)}{(0.15)}$

$= 31.9 \ W/m^2 \cdot °C$

b) $h = Nu \frac{k}{L} = (167)\frac{(0.0283)}{(0.15)}$

$= 31.5 \ W/m^2 \cdot °C$

STEP ⑦ : $T_s = ?$

$\dot{Q} = hA\,(T_s - T_\infty)$

$\therefore T_s = T_\infty + \frac{\dot{Q}}{hA}$

a) $T_s = 50 + \frac{15}{(31.9)(0.15 \times 0.15)}$

$= 70.9 \ °C$

b) $T_s = 50 + \frac{15}{(31.5)(0.15 \times 0.15)}$

$= 71.2 \ °C$

T_f CHECK...

$T_f = \frac{T_s + T_\infty}{2}$

$= \frac{71 + 50}{2}$

$= 60.5 \ °C$

(CLOSE TO OUR GUESS).

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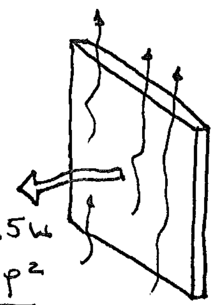
c) NATURAL CONVECTION

STEP ①: $L = 0.15 \ m$

GEOMETRY Ⓔ, VERTICAL SURFACE.



15W

$T_\infty = 50 °C$

$T_f = 60 °C$ (ASSUMED)

$T_s = 70 °C$ (ASSUMED).

STEP ②: NATURAL CONVECTION

STEP ③: DONE

STEP ④: $Gr = \frac{g\beta (T_s - T_\infty) L^3 \rho^2}{\mu^2}$

$= \frac{(9.81)(0.003)(70-50)(0.15)^3(1.079)^2}{(1.99 \times 10^{-5})^2}$

$\beta = \frac{1}{60 + 273} = 0.003$

$= 5.84 \times 10^6$

STEP ⑤: $Gr\,Pr = (5.84 \times 10^6)(0.708) = 4.135 \times 10^6$

$\therefore \quad Nu = C\,(Gr\,Pr)^n$

$= 0.555\,(4.135 \times 10^6)^{0.25}$

$= 25.03$

STEP ⑥: $h = Nu \frac{k}{L} = (25.03)\left(\frac{0.0283}{0.15}\right)$

$= 4.72 \ W/m^2 \cdot °C$

STEP ⑦: $T_s = T_\infty + \frac{\dot{Q}}{hA} = 50 + \frac{15}{(4.72)(0.15 \times 0.15)}$

$\underline{T_s = 191.2 °C}$

ESTIMATES A BIT LOW I'D SAY

TABLE A-18

Properties of gases at 1 atm pressure

Temperature, T K	Density, ρ kg/m^3	Specific heat, C_p J/kg · °C	Thermal conductivity, k W/m · °C	Thermal diffusivity, α m^2/s	Dynamic viscosity, μ kg/m · s	Kinematic viscosity, ν m^2/s	Prandtl number, Pr
Air							
200	1.766	1003	0.0181	1.02×10^{-5}	1.34×10^{-5}	0.76×10^{-5}	0.740
250	1.413	1003	0.0223	1.57×10^{-5}	1.61×10^{-5}	1.14×10^{-5}	0.724
280	1.271	1004	0.0246	1.95×10^{-5}	1.75×10^{-5}	1.40×10^{-5}	0.717
290	1.224	1005	0.0253	2.08×10^{-5}	1.80×10^{-5}	1.48×10^{-5}	0.714
298	1.186	1005	0.0259	2.18×10^{-5}	1.84×10^{-5}	1.55×10^{-5}	0.712
300	1.177	1005	0.0261	2.21×10^{-5}	1.85×10^{-5}	1.57×10^{-5}	0.712
310	1.143	1006	0.0268	2.35×10^{-5}	1.90×10^{-5}	1.67×10^{-5}	0.711
320	1.110	1006	0.0275	2.49×10^{-5}	1.94×10^{-5}	1.77×10^{-5}	0.710
330	1.076	1007	0.0283	2.64×10^{-5}	1.99×10^{-5}	1.86×10^{-5}	0.708
340	1.043	1007	0.0290	2.78×10^{-5}	2.03×10^{-5}	1.96×10^{-5}	0.707
350	1.009	1008	0.0297	2.92×10^{-5}	2.08×10^{-5}	2.06×10^{-5}	0.706
400	0.883	1013	0.0331	3.70×10^{-5}	2.29×10^{-5}	2.60×10^{-5}	0.703
450	0.785	1020	0.0363	4.54×10^{-5}	2.49×10^{-5}	3.18×10^{-5}	0.700
500	0.706	1029	0.0395	5.44×10^{-5}	2.68×10^{-5}	3.80×10^{-5}	0.699
550	0.642	1039	0.0426	6.39×10^{-5}	2.86×10^{-5}	4.45×10^{-5}	0.698
600	0.589	1051	0.0456	7.37×10^{-5}	3.03×10^{-5}	5.15×10^{-5}	0.698
700	0.504	1075	0.0513	9.46×10^{-5}	3.35×10^{-5}	6.64×10^{-5}	0.702
800	0.441	1099	0.0569	11.7×10^{-5}	3.64×10^{-5}	8.25×10^{-5}	0.704
900	0.392	1120	0.0625	14.2×10^{-5}	3.92×10^{-5}	9.99×10^{-5}	0.705
1000	0.353	1141	0.0672	16.7×10^{-5}	4.18×10^{-5}	11.8×10^{-5}	0.709
1200	0.294	1175	0.0759	22.2×10^{-5}	4.65×10^{-5}	15.8×10^{-5}	0.720
1400	0.252	1201	0.0835	27.6×10^{-5}	5.09×10^{-5}	20.2×10^{-5}	0.732
1600	0.221	1240	0.0904	33.0×10^{-5}	5.49×10^{-5}	24.9×10^{-5}	0.753
1800	0.196	1276	0.0970	38.3×10^{-5}	5.87×10^{-5}	29.9×10^{-5}	0.772
2000	0.177	1327	0.1032	44.1×10^{-5}	6.23×10^{-5}	35.3×10^{-5}	0.801
Ammonia (NH_3)							
200	1.038	2199	0.0153	0.67×10^{-5}	6.89×10^{-6}	0.66×10^{-5}	0.990
250	0.831	2248	0.0197	1.05×10^{-5}	8.53×10^{-6}	1.03×10^{-5}	0.973
300	0.692	2298	0.0246	1.55×10^{-5}	10.27×10^{-6}	1.48×10^{-5}	0.959
350	0.593	2349	0.0302	2.17×10^{-5}	12.06×10^{-6}	2.03×10^{-5}	0.938
400	0.519	2402	0.0364	2.92×10^{-5}	13.90×10^{-6}	2.68×10^{-5}	0.917
450	0.461	2455	0.0433	3.82×10^{-5}	15.76×10^{-6}	3.42×10^{-5}	0.894
500	0.415	2507	0.0506	4.86×10^{-5}	17.63×10^{-6}	4.25×10^{-5}	0.873
550	0.378	2559	0.0580	6.00×10^{-5}	19.5×10^{-6}	5.16×10^{-5}	0.860
600	0.346	2611	0.0656	7.26×10^{-5}	21.4×10^{-6}	6.18×10^{-5}	0.852
700	0.297	2710	0.0811	10.1×10^{-5}	25.1×10^{-6}	8.45×10^{-5}	0.839
800	0.260	2810	0.0977	13.4×10^{-5}	28.8×10^{-6}	11.1×10^{-5}	0.828
Argon							
200	2.435	523.6	0.0124	0.98×10^{-5}	1.60×10^{-5}	0.66×10^{-5}	0.674
250	1.948	522.2	0.0152	1.49×10^{-5}	1.95×10^{-5}	1.00×10^{-5}	0.672
300	1.623	521.6	0.0177	2.09×10^{-5}	2.27×10^{-5}	1.40×10^{-5}	0.669
350	1.392	521.2	0.0201	2.78×10^{-5}	2.57×10^{-5}	1.85×10^{-5}	0.666
400	1.218	521.0	0.0223	3.52×10^{-5}	2.85×10^{-5}	2.34×10^{-5}	0.665
450	1.082	520.9	0.0244	4.33×10^{-5}	3.12×10^{-5}	2.88×10^{-5}	0.665