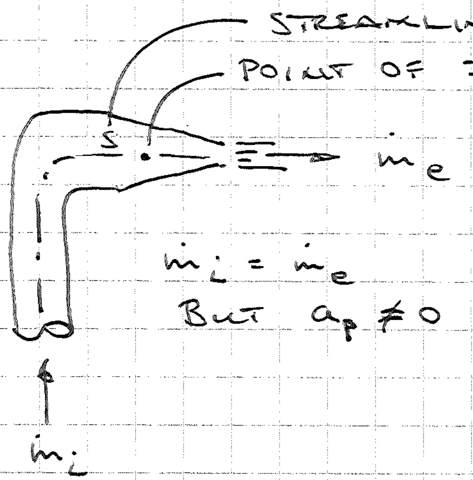
E292

WHERE DOES BERNOULLI EQUATION COME FROM?

IN SOME WAYS IT IS SIMPLY AN EXTENSION OF $F = ma$.
THAT MEANS THE EQ^{N} LOOKS AT FORCES, MASS AND ACCELERATION OF FLOW.

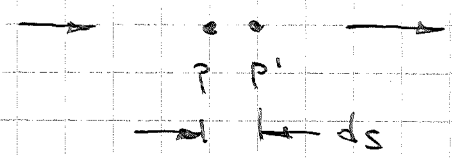
CONSIDER THIS "STEADY FLOW SITUATION"



$\dot{m}_i = \dot{m}_e$
BUT $a_p \neq 0$

IF YOU OBSERVE V_p ... IT IS CONSTANT AT ANY TIME OBSERVED.

HOWEVER ...



$V_p \neq V_{p'}$

$\therefore V_p \neq$ CONSTANT RELATIVE TO S.

$\therefore a_p \neq 0$

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt$$

$$\div dt: \quad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

WHEN $\dot{m}_i = \dot{m}_e$ (STEADY FLOW).

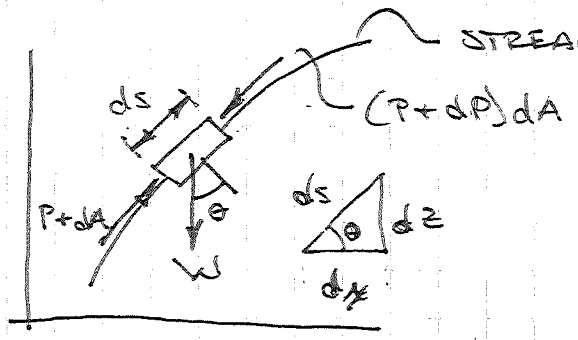
$\frac{\partial V}{\partial t} = 0$ SO VELOCITY ONLY CHANGES ALONG S.

SO $a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V$

THUS $\underline{a_s = V\frac{dV}{ds}}$ (FOR STEADY FLOW).

NOW CONSIDER A <u>SLUG</u> OF FLUID...

STREAMLINE, S (THE PATH).



USING: $F_s = ma_s$ & $F = PA$ (FROM $P = \frac{F}{A}$).

$$P\,dA - (P + dP)\,dA - \underset{\rho g \underbrace{dA\,ds}_{\text{VOLUME},\ dV}}{W}\ \underbrace{\sin\theta}_{\frac{dz}{ds}} = \underset{\underbrace{\rho\,dA\,ds}_{\rho(\text{VOLUME}) = m}}{m}\,V\underbrace{\frac{dV}{ds}}_{a_s}$$

SO $-dP\,dA - \rho g\,dA\,ds\,\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds}$

<u>$\div dA$</u> & <u>CANCEL ds's</u>.

$$-dP - \rho g\,dz = \rho V\,dV$$

<u>$\div \rho$</u> & FIX SIGNS.

$$\frac{dP}{\rho} + V\,dV + g\,dz = 0$$

NOW INTEGRATE

$$\int\frac{dP}{\rho} + \int V\,dV + \int g\,dz = 0$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{CONSTANT (ALONG STREAMLINE).}$$

∴ THAT'S STARTING TO LOOK RIGHT.

FOR STEADY FLOW ...

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

[UNITS: ENERGY → JOULES = N·m

SPECIFIC ENERGY → $J/kg = \frac{N \cdot m}{kg}$

$$\frac{N \cdot m}{kg} = \frac{\frac{kg \cdot m}{s^2} \times m}{kg} = \frac{\cancel{kg} \cdot m^2}{\cancel{kg} \cdot s^2} = \frac{m^2}{s^2}$$

$$gz = \frac{m}{s^2} \times m = \frac{m^2}{s^2}$$]

BUT THIS IS ONLY USEFUL FOR INVISCID (NO FRICTION) FLOW.
WHAT GIVES?

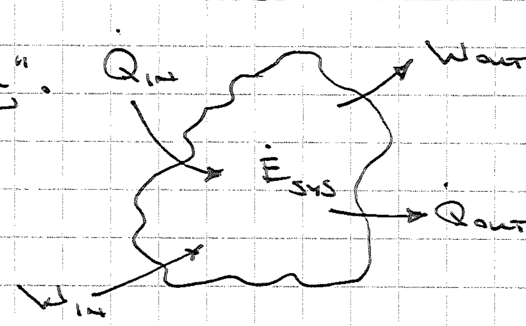
IT'S TIME TO BROADEN THINGS A BIT ...

GENERAL ENERGY EQUATION.

CLOSED (FIXED MASS).

FIRST, CONSIDER "THE SYSTEM".

WHAT IS THAT THING ?



$$\underbrace{\dot{Q}_{NET,IN}}_{\dot{Q}_{IN} - \dot{Q}_{OUT}} + \underbrace{\dot{W}_{NET,IN}}_{\dot{W}_{IN} - \dot{W}_{OUT}} = \frac{dE_{SYS}}{dt}$$

(FOOD RULE: IN IS GOOD ∴ POSITIVE)

E_{SYS} → JOULES.

e_{SYS} → J/kg (SPECIFIC ENERGY).

ALSO SYSTEM MASS, $m_{SYS} = \rho\, dV$

SO $$\dot{Q}_{NET,IN} + \dot{W}_{NET,IN} = \frac{d}{dt}\int_{SYS} \rho e\, dV$$

FROM THE PREVIOUS SECTION ... $e = u + KE + PE$

$$= u + \frac{V^2}{2} + gz \quad , \quad \frac{m^2}{s^2} = \frac{J}{kg} \;\therefore = \text{SPECIFIC ENERGY.}$$

NOTE: u = INTERNAL ENERGY

= TEMPERATURE RELATED MOLECULAR ENERGY.

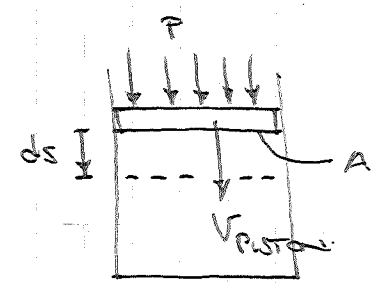
<u>HEAT TRANSFER, Q</u> → TRANSFER OF MOLECULAR KINETIC ENERGY
(DISCUSS).

TERM: $\boxed{\text{ADIABATIC}}$ MEANS: PERFECTLY INSULATED ($\therefore \dot{Q} = 0$).

<u>WORK TRANSFER, W</u> → FORCE ACTING THROUGH A DISTANCE.

$$W_{TOTAL} = W_{SHAFT} + W_{PRESSURE} + W_{VISCOUS} + W_{OTHER}$$

$$\dot{W}_{SHAFT} = \underset{rad/s}{\omega}\, \underset{N\text{-}m}{T_{SHAFT}} = 2\pi \underset{RPS}{n}\, \underset{N\text{-}m}{T_{SHAFT}}$$



$$\Delta \dot{W} = \underset{\text{WORK DONE AT A MOVING BOUNDARY BY PRESSURE}}{\Delta W_{BOUNDARY}} = \underset{F = PA}{PA}\, \underset{ds/dt}{V_{PISTON}}$$

or $d\dot{W} = P\,dA\,V_{PISTON}$

AND $W_{PRESSURE,\ NET\ IN.} = \int_A P V_A \, dA$

DIVIDING UP WORK FLOW INTO <u>SHAFT</u> & <u>PRESSURE</u>

$$\dot{W}_{NET,IN} = \dot{W}_{SHAFT,IN} + \underbrace{\dot{W}_{PRESSURE,IN}}_{\int_A P V \, dA}$$

FOR CLOSED SYSTEMS

SO NOW: $\dot{Q}_{NET,IN} + \dot{W}_{SHAFT,\,NET\,IN} + \dot{W}_{PRESSURE,\,NET\,IN} = \dfrac{dE_{sys}}{dt}$

THIS CAN BE GENERALIZED FOR <u>OPEN SYSTEMS</u> OR <u>CONTROL VOLUMES</u>

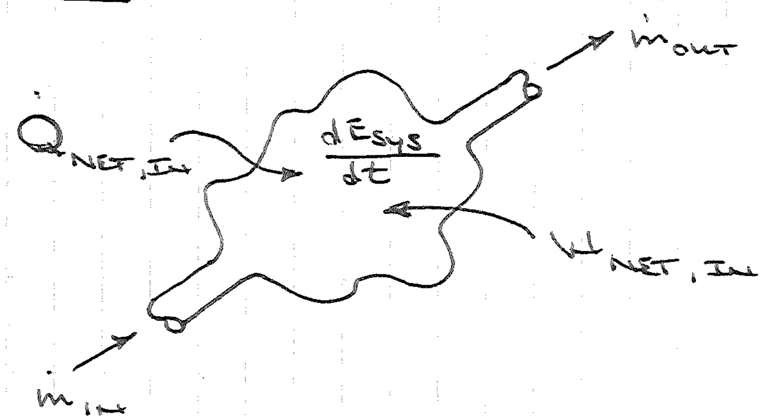
ENERGY CARRIED BY $\dot{m}$:

$$\dot{e}_{mass} = \int_{CS} e \underbrace{\rho V \, dA}_{\dot{m}}$$

$$\frac{kg}{m^3} \times \frac{m}{s} \times m^2 = kg/s$$

$$e = \underbrace{\left(u + \frac{V^2}{2} + gz\right)}_{\text{SPECIFIC ENERGY IN A kg OF MASS}}$$

SPECIFIC ENERGY IN A kg OF MASS



ALSO ... AS BEFORE

$$\frac{dE_{sys}}{dt} = \frac{d}{dt} \int_{CV} e \underbrace{\rho \, dV}_{m}$$

$$m, \quad \frac{kg}{m^3} \times m^3$$

$$\text{SO } \dot{Q}_{\text{NET, IN}} + \dot{W}_{\text{SHAFT, NET IN}} + \dot{W}_{\text{PRESSURE, NET IN}} + \int_{CS} e \, \underbrace{\rho V dA}_{} = \frac{d}{dt}\int_{CV} e \rho \, dV$$

$$\int_{CS} P V dA = \int \frac{P}{\rho} \times \underbrace{\rho V dA}_{\text{SIMILAR.}}$$

$$\text{THUS } \dot{Q}_{\text{NET, IN}} + \dot{W}_{\text{SHAFT, NET IN}} + \int_{CS} \left(\frac{P}{\rho} + e\right) \rho V dA = \frac{d}{dt}\int_{CV} e \rho \, dV$$

FLOW WORK TERM

IT TURNS OUT THAT $\frac{P}{\rho} = P\nu$

SPECIFIC VOLUME, $\frac{m^3}{kg}$

$$\frac{N/m^2}{kg/m^3} = \frac{N\text{-}m}{kg} \quad \ldots \quad N/m^2 \times \frac{m^3}{kg} = \frac{N-m}{kg}$$

$\ldots$ N-m IS AN ENERGY UNIT, JOULES.

$\frac{N-m}{kg}$ IS A SPECIFIC ENERGY UNIT, J/kg.

THAT MEANS ...

$$\int_{CS} \left(\frac{P}{\rho} + e\right) \rho V dA = \int_{CS} (P\nu + e) \underbrace{\rho V dA}_{d\dot{m}}$$

INTEGRATING OVER THE SURFACE (AT THE OPEN BITS). ALL THIS BECOMES.

$$\sum_{\text{MASS FLOW, NET IN}} \dot{m} (P\nu + e) = \sum_{m, \text{NET IN}} \dot{m} \left(P\nu + u + \frac{V^2}{2} + gz\right)$$

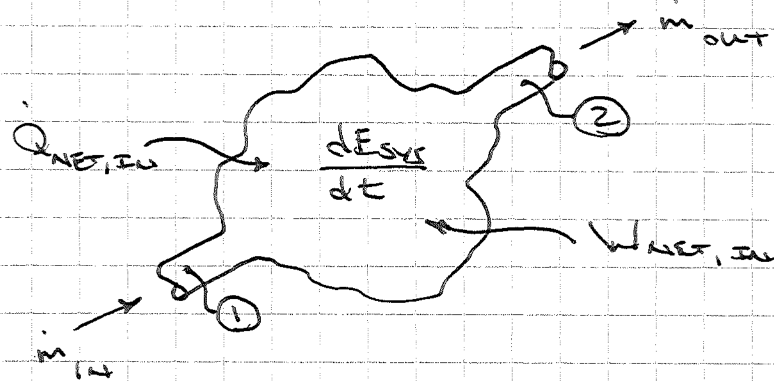
YOU KNOW THAT

ENTHALPY, $\boxed{h = u + pV}$

THUS AT LONG LAST WE HAVE

$$\dot{Q}_{NET,IN} + \dot{W}_{SHAFT,IN} + \sum_{\substack{MASS \\ FLOW}, NET\ IN} \dot{m}\left(h + \frac{V^2}{2} + gz\right) = \frac{dE_{SYS}}{dt}\left[= \frac{d}{dt}\int_{CV} e\rho\, dV\right]$$

FOR A SIMPLE SYSTEM AT STEADY STATE ...



STEADY STATE MEANS ...

$$\dot{m}_{IN} = \dot{m}_{OUT} = \dot{m}$$

$$\frac{dE_{SYS}}{dt} = 0$$

SO $\dot{Q}_{NET,IN} + \dot{W}_{SHAFT,IN} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right] = 0$

OR PER kg (SPECIFIC BASIS)

$$\boxed{\dot{q}_{NET,IN} + \dot{w}_{SHAFT,IN} + \left[(h_1 - h_2) + \frac{V_1^2 - V_2}{2} + g(z_1 - z_2)\right] = 0}$$