SIMILITUDE EXAMPLE - EXPANDED

Ex 8.3

$$Re_m = Re_p \qquad Re = \frac{VLP}{\mu}$$

$$\frac{V_m L_m \rho_m}{\mu_m} = \frac{V_p L_p \rho_p}{\mu_p}$$

$$\text{so } V_m = V_p \left(\frac{L_p}{L_m}\right)\left(\frac{\rho_p}{\rho_m}\right)^{1}\left(\frac{\mu_m}{\mu_p}\right)^{1}$$

Ex 8.4

$$Re_m = Re_p \qquad \text{NOTE} \quad \nu = \frac{\mu}{\rho}$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\text{so } \frac{V_m}{V_p} = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p} \qquad \text{NOTE: } Q = VA \qquad A = \frac{\pi D^2}{4} = \frac{\pi L^2}{4}$$

$$\text{so } \frac{Q_m}{Q_p} = \frac{V_m A_m}{V_p A_p} = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}\frac{A_m}{A_p} \qquad \text{ALSO NOTE: } \frac{A_m}{A_p} = \frac{L_m^2}{L_p^2}$$

$$\text{so } \frac{Q_m}{Q_p} = \frac{L_m^2}{L_p^2}\frac{L_p}{L_m}\frac{\nu_m}{\nu_p} \quad ; \quad \text{IF } \nu_m = \nu_p$$

$$\frac{Q_m}{Q_p} = \frac{L_m}{L_p}$$

$$\therefore Q_m = Q_p\left(\frac{L_m}{L_p}\right)$$

Ex. 8.5 (BASED ON 8.3).

$$\frac{\Delta p_m}{\frac{1}{2}\rho_m V_m^2} = \frac{\Delta p_p}{\frac{1}{2}\rho_p V_p^2}$$

$$\text{so } \Delta p_p = \Delta p_m \frac{\rho_p}{\rho_m}^{1} \frac{V_p^2}{V_m^2}$$

FROM Ex 8.3 $\longrightarrow$ $Re_m = Re_p \longrightarrow \frac{V_p}{V_m} = \frac{L_p}{L_m}$

$$\text{so } \Delta p_p = \Delta p_m \left(\frac{L_p}{L_m}\right)^2$$

Ex. 8.6 (BASED ON Ex 8.3).

FROM $\frac{\Delta p_m}{\frac{1}{2}\rho_m V_m^2} = \frac{\Delta p_p}{\frac{1}{2}\rho_p V_p^2}$ AND $Re_p = Re_m$

$$\frac{\Delta p_p}{\Delta p_m} = \frac{\rho_p}{\rho_m}^{1} \frac{V_p^2}{V_m^2}$$

$$\frac{V_p L_p}{\nu_p} = \frac{V_m L_m}{\nu_m}$$

$$\text{so } \frac{V_p}{V_m} = \frac{\nu_p}{\nu_m}^{1} \frac{L_m}{L_p}$$

$$\text{so } \frac{\Delta p_p}{\Delta p_m} = \frac{L_m^2}{L_p^2}$$

NOW THIS IS TRICKY ...

X BOTH SIDES BY: $\frac{L_p^2}{L_m^2} \left(= \frac{A_p}{A_m}\right)$

$$\frac{\Delta p_p}{\Delta p_m} \times \frac{L_p^2}{L_m^2} = \frac{L_m^2}{L_p^2} \times \frac{L_p^2}{L_m^2} = 1$$

$$\text{so } \frac{\Delta p_p}{\Delta p_m} \times \frac{A_p}{A_m} = \frac{F_{Dp}}{F_{Dm}} = 1 \qquad \because F = pA$$

Ex 8.8

$$F_{r_m} = F_{r_p} \qquad F_r = \frac{V}{\sqrt{gL}}$$

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \qquad \because g_m = g_p$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \qquad \text{RECALL: } Q = VA$$

$$\text{so } \frac{V_m}{V_p} \times \frac{A_m}{A_p} = \frac{A_m}{A_p} \times \sqrt{\frac{L_m}{L_p}} = \frac{Q_m}{Q_p} \qquad \text{NOTE: } \frac{A_m}{A_p} = \frac{L_m^2}{L_p^2}$$

$$\text{so } \frac{Q_m}{Q_p} = \frac{L_m^2}{L_p^2}\sqrt{\frac{L_m}{L_p}} \quad \Big\} \rightarrow \left[a^2 \times a^{1/2} = a^{(2+1/2)} = a^{5/2}\right]$$

$$\therefore \frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^{5/2}$$