21–1 ▪ INTRODUCTION

Consider a hot object that is suspended in an evacuated chamber whose walls are at room temperature (Fig. 21–1). The hot object will eventually cool down and reach thermal equilibrium with its surroundings. That is, it will lose heat until its temperature reaches the temperature of the walls of the chamber. Heat transfer between the object and the chamber could not have taken place by conduction or convection, because these two mechanisms cannot occur in a vacuum. Therefore, heat transfer must have occurred through another mechanism that involves the emission of the internal energy of the object. This mechanism is *radiation.*

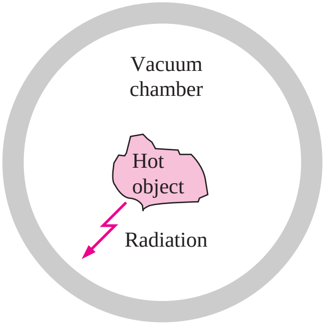


FIGURE 21–1
A hot object in a vacuum chamber loses heat by radiation only.

Radiation differs from the other two heat transfer mechanisms in that it does not require the presence of a material medium to take place. In fact, energy transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a *vacuum.* Also, radiation transfer occurs in solids as well as liquids and gases. In most practical applications, all three modes of heat transfer occur concurrently at varying degrees. But heat transfer through an evacuated space can occur only by radiation. For example, the energy of the sun reaches the earth by radiation.

You will recall that heat transfer by conduction or convection takes place in the direction of decreasing temperature; that is, from a high-temperature medium to a lower-temperature one. It is interesting that radiation heat transfer can occur between two bodies separated by a medium colder than both bodies (Fig. 21–2). For example, solar radiation reaches the surface of the earth after passing through cold air layers at high altitudes. Also, the radiation-absorbing surfaces inside a greenhouse reach high temperatures even when its plastic or glass cover remains relatively cool.

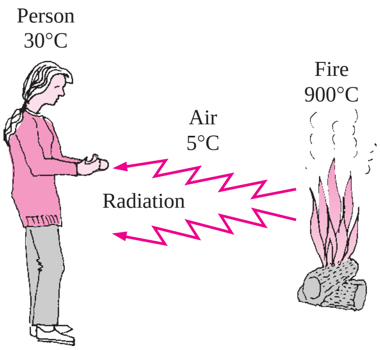


FIGURE 21–2
Unlike conduction and convection, heat transfer by radiation can occur between two bodies, even when they are separated by a medium colder than both of them.

The theoretical foundation of radiation was established in 1864 by physicist James Clerk Maxwell, who postulated that accelerated charges or changing electric currents give rise to electric and magnetic fields. These rapidly moving fields are called **electromagnetic waves** or **electromagnetic radiation,** and they represent the energy emitted by matter as a result of the changes in the electronic configurations of the atoms or molecules. In 1887, Heinrich Hertz experimentally demonstrated the existence of such waves. Electromagnetic waves transport energy just like other waves, and all electromagnetic waves travel at the *speed of light* in a vacuum, which is $C_0 = 2.9979 \times 10^8$ m/s. Electromagnetic waves are characterized by their *frequency* υ or *wavelength* λ. These two properties in a medium are related by

$$\lambda = \frac{c}{\upsilon} \tag{21–1}$$

where c is the speed of propagation of a wave in that medium. The speed of propagation in a medium is related to the speed of light in a vacuum by $c = c_0/n$, where n is the *index of refraction* of that medium. The refractive index is essentially unity for air and most gases, about 1.5 for glass, and about 1.33 for water. The commonly used unit of wavelength is the *micrometer* (μm) or micron, where 1 μm $= 10^{-6}$ m. Unlike the wavelength and the speed of propagation, the frequency of an electromagnetic wave depends only on the source and is independent of the medium through which the wave travels. The *frequency* (the number of oscillations per second) of an electromagnetic wave can range from less than a million Hz to a septillion Hz or higher, depending on the source. Note from Eq. 21–1 that the wavelength and the frequency of electromagnetic radiation are inversely proportional.

It has proven useful to view electromagnetic radiation as the propagation of a collection of discrete packets of energy called **photons** or **quanta,** as proposed by Max Planck in 1900 in conjunction with his *quantum theory.* In this view, each photon of frequency υ is considered to have an energy of

$$e = h\upsilon = \frac{hc}{\lambda} \tag{21–2}$$

where $h = 6.6256 \times 10^{-34}$ J · s is *Planck's constant.* Note from the second part of Eq. 21–2 that the energy of a photon is inversely proportional to its wavelength. Therefore, shorter-wavelength radiation possesses larger photon energies. It is no wonder that we try to avoid very-short-wavelength radiation such as gamma rays and X-rays since they are highly destructive.

21–2 ▪ THERMAL RADIATION

Although all electromagnetic waves have the same general features, waves of different wavelength differ significantly in their behavior. The electromagnetic radiation encountered in practice covers a wide range of wavelengths, varying from less than 10^{-10} μm for cosmic rays to more than 10^{10} μm for electrical power waves. The **electromagnetic spectrum** also includes gamma rays, X-rays, ultraviolet radiation, visible light, infrared radiation, thermal radiation, microwaves, and radio waves, as shown in Fig. 21–3.

Different types of electromagnetic radiation are produced through various mechanisms. For example, *gamma rays* are produced by nuclear reactions, *X-rays* by the bombardment of metals with high-energy electrons, *microwaves* by special types of electron tubes such as klystrons and magnetrons, and *radio waves* by the excitation of some crystals or by the flow of alternating current through electric conductors.

The short-wavelength gamma rays and X-rays are primarily of concern to nuclear engineers, while the long-wavelength microwaves and radio waves are of concern to electrical engineers. The type of electromagnetic radiation that is pertinent to heat transfer is the **thermal radiation** emitted as a result of energy transitions of molecules, atoms, and electrons of a substance. Temperature is a measure of the strength of these activities at the microscopic level, and the rate of thermal radiation emission increases with increasing temperature. Thermal radiation is continuously emitted by all matter whose temperature is above absolute zero. That is, everything around us such as walls, furniture, and our friends constantly emits (and absorbs) radiation (Fig. 21–4). Thermal radiation is also defined as the portion of the electromagnetic spectrum that extends from about 0.1 to 100 μm, since the radiation emitted by bodies due to their temperature falls almost entirely into this wavelength range. Thus, thermal radiation includes the entire visible and infrared (IR) radiation as well as a portion of the ultraviolet (UV) radiation.

What we call **light** is simply the *visible* portion of the electromagnetic spectrum that lies between 0.40 and 0.76 μm. Light is characteristically no different than other electromagnetic radiation, except that it happens to trigger the

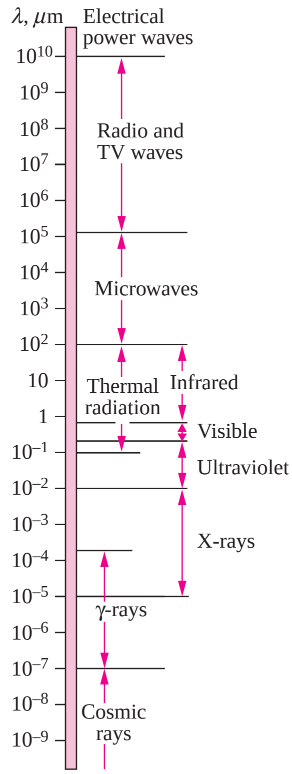


FIGURE 21–3
The electromagnetic wave spectrum.

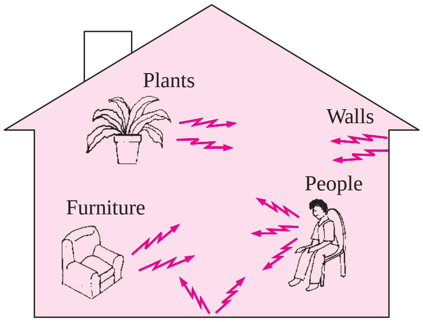


FIGURE 21–4
Everything around us constantly emits thermal radiation.

TABLE 21–1

The wavelength ranges of different colors

Color	Wavelength band
Violet	0.40–0.44 μm
Blue	0.44–0.49 μm
Green	0.49–0.54 μm
Yellow	0.54–0.60 μm
Orange	0.60–0.67 μm
Red	0.63–0.76 μm

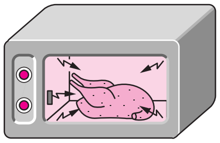


FIGURE 21–5
Food is heated or cooked in a microwave oven by absorbing the electromagnetic radiation energy generated by the magnetron of the oven.

sensation of seeing in the human eye. Light, or the visible spectrum, consists of narrow bands of color from violet (0.40–0.44 μm) to red (0.63–0.76 μm), as shown in Table 21–1.

A body that emits some radiation in the visible range is called a light source. The sun is obviously our primary light source. The electromagnetic radiation emitted by the sun is known as **solar radiation,** and nearly all of it falls into the wavelength band 0.3–3 μm. Almost *half* of solar radiation is light (i.e., it falls into the visible range), with the remaining being ultraviolet and infrared.

The radiation emitted by bodies at room temperature falls into the **infrared** region of the spectrum, which extends from 0.76 to 100 μm. Bodies start emitting noticeable visible radiation at temperatures above 800 K. The tungsten filament of a lightbulb must be heated to temperatures above 2000 K before it can emit any significant amount of radiation in the visible range.

The **ultraviolet** radiation includes the low-wavelength end of the thermal radiation spectrum and lies between the wavelengths 0.01 and 0.40 μm. Ultraviolet rays are to be avoided since they can kill microorganisms and cause serious damage to humans and other living organisms. *About 12 percent of solar radiation is in the ultraviolet range,* and it would be devastating if it were to reach the surface of the earth. Fortunately, the ozone (O_3) layer in the atmosphere acts as a protective blanket and absorbs most of this ultraviolet radiation. The ultraviolet rays that remain in sunlight are still sufficient to cause serious sunburns to sun worshippers, and prolonged exposure to direct sunlight is the leading cause of skin cancer, which can be lethal. Recent discoveries of "holes" in the ozone layer have prompted the international community to ban the use of ozone-destroying chemicals such as the refrigerant Freon-12 in order to save the earth. Ultraviolet radiation is also produced artificially in fluorescent lamps for use in medicine as a bacteria killer and in tanning parlors as an artificial tanner. The connection between skin cancer and ultraviolet rays has caused dermatologists to issue strong warnings against its use for tanning.

Microwave ovens utilize electromagnetic radiation in the **microwave** region of the spectrum generated by microwave tubes called *magnetrons.* Microwaves in the range of 10^2–10^5 μm are very suitable for use in cooking since they are *reflected* by metals, *transmitted* by glass and plastics, and *absorbed* by food (especially water) molecules. Thus, the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food. The fast and efficient cooking of microwave ovens has made them one of the essential appliances in modern kitchens (Fig. 21–5).

Radars and cordless telephones also use electromagnetic radiation in the microwave region. The wavelength of the electromagnetic waves used in radio and TV broadcasting usually ranges between 1 and 1000 m in the **radio wave** region of the spectrum.

In heat transfer studies, we are interested in the energy emitted by bodies because of their temperature only. Therefore, we will limit our consideration to *thermal radiation,* which we will simply call *radiation.* The relations developed below are restricted to thermal radiation only and may not be applicable to other forms of electromagnetic radiation.

The electrons, atoms, and molecules of all solids, liquids, and gases above absolute zero temperature are constantly in motion, and thus radiation is constantly emitted, as well as being absorbed or transmitted throughout the entire volume of matter. That is, radiation is a **volumetric phenomenon.** However, for opaque (nontransparent) solids such as metals, wood, and rocks, radiation is considered to be a **surface phenomenon,** since the radiation emitted by the interior regions can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface (Fig. 21–6). Note that the radiation characteristics of surfaces can be changed completely by applying thin layers of coatings on them.

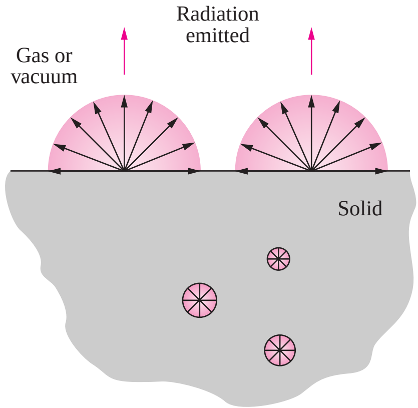


FIGURE 21–6
Radiation in opaque solids is considered a surface phenomenon since the radiation emitted only by the molecules at the surface can escape the solid.

21–3 ▪ BLACKBODY RADIATION

A body at a temperature above absolute zero emits radiation in all directions over a wide range of wavelengths. The amount of radiation energy emitted from a surface at a given wavelength depends on the material of the body and the condition of its surface as well as the surface temperature. Therefore, different bodies may emit different amounts of radiation per unit surface area, even when they are at the same temperature. Thus, it is natural to be curious about the *maximum* amount of radiation that can be emitted by a surface at a given temperature. Satisfying this curiosity requires the definition of an idealized body, called a *blackbody,* to serve as a standard against which the radiative properties of real surfaces may be compared.

A **blackbody** is defined as *a perfect emitter and absorber of radiation.* At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission (Fig. 21–7). That is, a blackbody is a *diffuse* emitter. The term *diffuse* means "independent of direction."

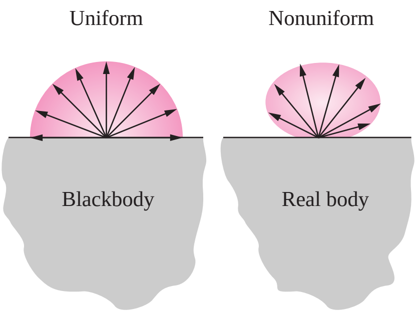


FIGURE 21–7
A blackbody is said to be a *diffuse* emitter since it emits radiation energy uniformly in all directions.

The radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 and expressed as

$$E_b(T) = \sigma T^4 \qquad (\text{W/m}^2) \tag{21–3}$$

where $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$ is the *Stefan–Boltzmann constant* and T is the absolute temperature of the surface in K. This relation was theoretically verified in 1884 by Ludwig Boltzmann. Equation 21–3 is known as the **Stefan–Boltzmann law** and E_b is called the **blackbody emissive power.** Note that the emission of thermal radiation is proportional to the *fourth power* of the absolute temperature.

Although a blackbody would appear *black* to the eye, a distinction should be made between the idealized blackbody and an ordinary black surface. Any surface that absorbs light (the visible portion of radiation) would appear black to the eye, and a surface that reflects it completely would appear white. Considering that visible radiation occupies a very narrow band of the spectrum from 0.4 to 0.76 μm, we cannot make any judgments about the blackness of a surface on the basis of visual observations. For example, snow and white paint reflect light and thus appear white. But they are essentially black for infrared radiation since they strongly absorb long-wavelength radiation. Surfaces coated with lampblack paint approach idealized blackbody behavior.

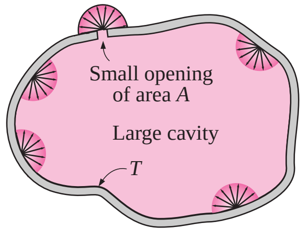


FIGURE 21–8
A large isothermal cavity at temperature T with a small opening of area A closely resembles a blackbody of surface area A at the same temperature.

Another type of body that closely resembles a blackbody is a *large cavity with a small opening,* as shown in Fig. 21–8. Radiation coming in through the opening of area A will undergo multiple reflections, and thus it will have several chances to be absorbed by the interior surfaces of the cavity before any part of it can possibly escape. Also, if the surface of the cavity is isothermal at temperature T, the radiation emitted by the interior surfaces will stream through the opening after undergoing multiple reflections, and thus it will have a diffuse nature. Therefore, the cavity will act as a perfect absorber and perfect emitter, and the opening will resemble a blackbody of surface area A at temperature T, regardless of the actual radiative properties of the cavity.

The Stefan–Boltzmann law in Eq. 21–3 gives the *total* blackbody emissive power E_b, which is the sum of the radiation emitted over all wavelengths. Sometimes we need to know the **spectral blackbody emissive power,** which is *the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area, and per unit wavelength about the wavelength* λ. For example, we are more interested in the amount of radiation an incandescent lightbulb emits in the visible wavelength spectrum than we are in the total amount emitted.

The relation for the spectral blackbody emissive power $E_{b\lambda}$ was developed by Max Planck in 1901 in conjunction with his famous quantum theory. This relation is known as **Planck's law** and is expressed as

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \qquad (\text{W/m}^2 \cdot \mu\text{m}) \qquad \textbf{(21–4)}$$

where

$$C_1 = 2\pi hc_0^2 = 3.742 \times 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2$$
$$C_2 = hc_0/k = 1.439 \times 10^4 \ \mu\text{m} \cdot \text{K}$$

Also, T is the absolute temperature of the surface, λ is the wavelength of the radiation emitted, and $k = 1.38065 \times 10^{-23}$ J/K is *Boltzmann's constant.* This relation is valid for a surface in a *vacuum* or a *gas.* For other mediums, it needs to be modified by replacing C_1 by C_1/n^2, where n is the index of refraction of the medium. Note that the term *spectral* indicates dependence on wavelength.

The variation of the spectral blackbody emissive power with wavelength is plotted in Fig. 21–9 for selected temperatures. Several observations can be made from this figure:

1. The emitted radiation is a continuous function of *wavelength.* At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.
2. At any wavelength, the amount of emitted radiation *increases* with increasing temperature.
3. As temperature increases, the curves shift to the left to the shorter-wavelength region. Consequently, a larger fraction of the radiation is emitted at *shorter wavelengths* at higher temperatures.
4. The radiation emitted by the *sun,* which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes. On the other hand, surfaces at $T \leq 800$ K emit almost entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other sources.

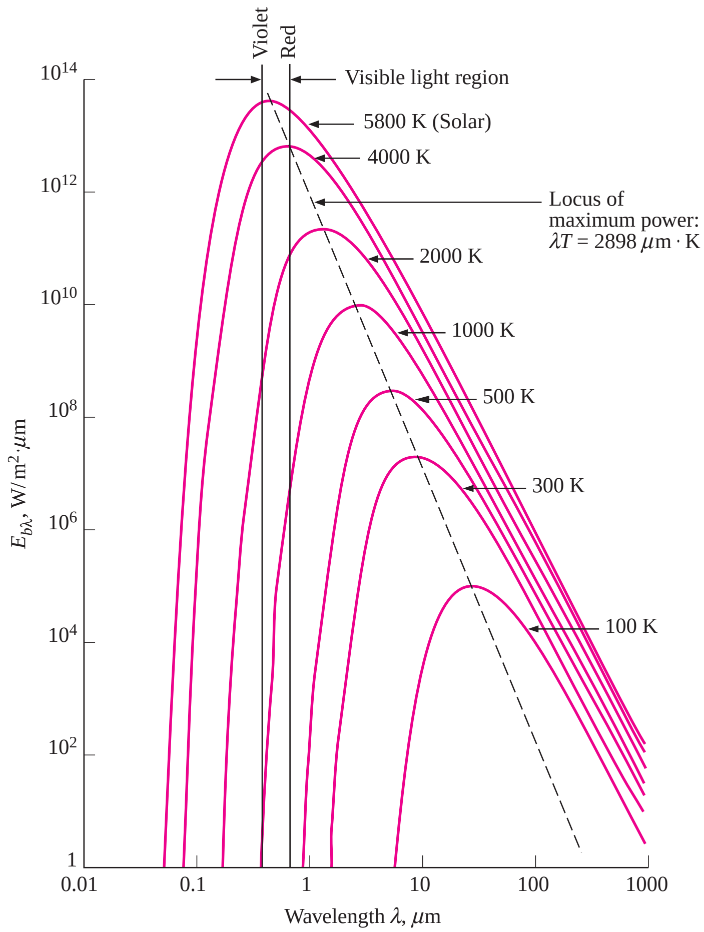


FIGURE 21–9
The variation of the blackbody emissive power with wavelength for several temperatures.

As the temperature increases, the peak of the curve in Fig. 21–9 shifts toward shorter wavelengths. The wavelength at which the peak occurs for a specified temperature is given by **Wien's displacement law** as

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m} \cdot \text{K} \qquad \textbf{(21–5)}$$

This relation was originally developed by Willy Wien in 1894 using classical thermodynamics, but it can also be obtained by differentiating Eq. 21–4 with respect to λ while holding T constant and setting the result equal to zero. A plot of Wien's displacement law, which is the locus of the peaks of the radiation emission curves, is also given in Fig. 21–9.

The peak of the solar radiation, for example, occurs at $\lambda = 2897.8/5780 = 0.50\ \mu$m, which is near the middle of the visible range. The peak of the radiation emitted by a surface at room temperature ($T = 298$ K) occurs at 9.72 μm, which is well into the infrared region of the spectrum.

An electrical resistance heater starts radiating heat soon after it is plugged in, and we can feel the emitted radiation energy by holding our hands facing the heater. But this radiation is entirely in the infrared region and thus cannot be sensed by our eyes. The heater would appear dull red when its temperature reaches about 1000 K, since it will start emitting a detectable amount (about 1 W/m^2 · μm) of visible red radiation at that temperature. As the temperature rises even more, the heater appears bright red and is said to be *red hot.* When the temperature reaches about 1500 K, the heater emits enough radiation in the entire visible range of the spectrum to appear almost *white* to the eye, and it is called *white hot.*

Although it cannot be sensed directly by the human eye, infrared radiation can be detected by infrared cameras, which transmit the information to microprocessors to display visual images of objects at night. *Rattlesnakes* can sense the infrared radiation or the "body heat" coming off warm-blooded animals, and thus they can see at night without using any instruments. Similarly, honeybees are sensitive to ultraviolet radiation. A surface that reflects all of the light appears *white*, while a surface that absorbs all of the light incident on it appears black. (Then how do we see a black surface?)

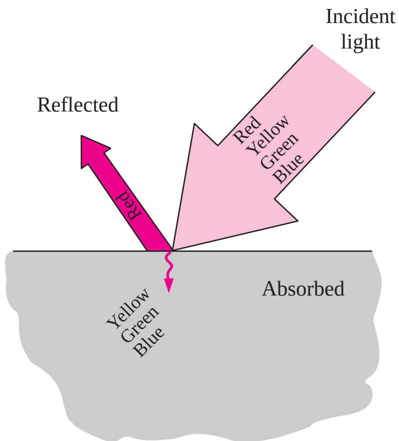


FIGURE 21–10
A surface that reflects red while absorbing the remaining parts of the incident light appears red to the eye.

It should be clear from this discussion that the color of an object is not due to emission, which is primarily in the infrared region, unless the surface temperature of the object exceeds about 1000 K. Instead, the color of a surface depends on the absorption and reflection characteristics of the surface and is due to selective absorption and reflection of the incident visible radiation coming from a light source such as the sun or an incandescent lightbulb. A piece of clothing containing a pigment that reflects red while absorbing the remaining parts of the incident light appears "red" to the eye (Fig. 21–10). Leaves appear "green" because their cells contain the pigment chlorophyll, which strongly reflects green while absorbing other colors.

It is left as an exercise to show that integration of the *spectral* blackbody emissive power $E_{b\lambda}$ over the entire wavelength spectrum gives the *total* blackbody emissive power E_b:

$$E_b(T) = \int_0^\infty E_{b\lambda}(\lambda, T)\, d\lambda = \sigma T^4 \qquad (\text{W/m}^2) \tag{21–6}$$

Thus, we obtained the Stefan–Boltzmann law (Eq. 21–3) by integrating Planck's law (Eq. 21–4) over all wavelengths. Note that on an $E_{b\lambda}$–λ chart, $E_{b\lambda}$ corresponds to any value on the curve, whereas E_b corresponds to the area under the entire curve for a specified temperature (Fig. 21–11). Also, the term *total* means "integrated over all wavelengths."

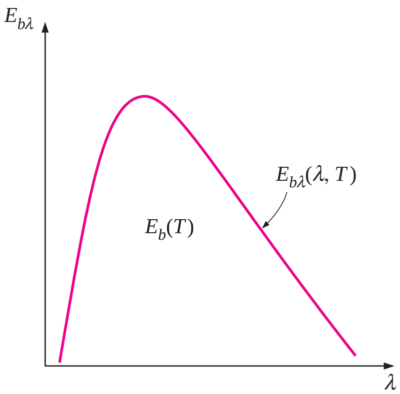


FIGURE 21–11
On an $E_{b\lambda}$–λ chart, the area under a curve for a given temperature represents the total radiation energy emitted by a blackbody at that temperature.

EXAMPLE 21–1 Radiation Emission from a Black Ball

Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Fig. 21–12. Assuming the ball closely approximates a blackbody, determine (*a*) the total blackbody emissive power, (*b*) the total amount of radiation emitted by the ball in 5 min, and (*c*) the spectral blackbody emissive power at a wavelength of 3 μm.

SOLUTION An isothermal sphere is suspended in air. The total blackbody emissive power, the total radiation emitted in 5 min, and the spectral blackbody emissive power at 3 μm are to be determined.

Assumptions The ball behaves as a blackbody.

Analysis (*a*) The total blackbody emissive power is determined from the Stefan–Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(800\ \text{K})^4 = \mathbf{23.2 \times 10^3\ W/m^2 = 23.2\ kW/m^2}$$

That is, the ball emits 23.2 kJ of energy in the form of electromagnetic radiation per second per m^2 of the surface area of the ball.

(*b*) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2\ \text{m})^2 = 0.1257\ \text{m}^2$$

$$\Delta t = (5\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 300\ \text{s}$$

$$Q_{\text{rad}} = E_b A_s\, \Delta t = (23.2\ \text{kW/m}^2)(0.1257\ \text{m}^2)(300\ \text{s})\left(\frac{1\ \text{kJ}}{1000\ \text{W} \cdot \text{s}}\right)$$

$$= \mathbf{876\ kJ}$$

That is, the ball loses 876 kJ of its internal energy in the form of electromagnetic waves to the surroundings in 5 min, which is enough energy to raise the temperature of 1 kg of water by 50°C. Note that the surface temperature of the ball cannot remain constant at 800 K unless there is an equal amount of energy flow to the surface from the surroundings or from the interior regions of the ball through some mechanisms such as chemical or nuclear reactions.

(*c*) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law to be

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.743 \times 10^8\ \text{W} \cdot \mu\text{m}^4/\text{m}^2}{(3\ \mu\text{m})^5\left[\exp\left(\frac{1.4387 \times 10^4\ \mu\text{m} \cdot \text{K}}{(3\ \mu\text{m})(800\ \text{K})}\right) - 1\right]}$$

$$= \mathbf{3848\ W/m^2 \cdot \mu m}$$

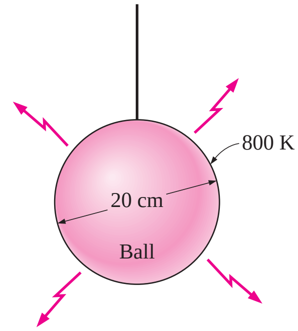


FIGURE 21–12
The spherical ball considered in Example 21–1.

The Stefan–Boltzmann law $E_b(T) = \sigma T^4$ gives the *total* radiation emitted by a blackbody at all wavelengths from $\lambda = 0$ to $\lambda = \infty$. But we are often interested in the amount of radiation emitted over *some wavelength band*. For example, an incandescent lightbulb is judged on the basis of the radiation it emits in the visible range rather than the radiation it emits at all wavelengths.

The radiation energy emitted by a blackbody per unit area over a wavelength band from $\lambda = 0$ to λ is determined from (Fig. 21–13)

$$E_{b,\,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(\lambda, T)\, d\lambda \qquad (\text{W/m}^2) \qquad \textbf{(21–7)}$$

It looks like we can determine $E_{b,\,0-\lambda}$ by substituting the $E_{b\lambda}$ relation from Eq. 21–4 and performing this integration. But it turns out that this integration does not have a simple closed-form solution, and performing a numerical integration each time we need a value of $E_{b,\,0-\lambda}$ is not practical. Therefore, we define a dimensionless quantity f_λ called the **blackbody radiation function** as

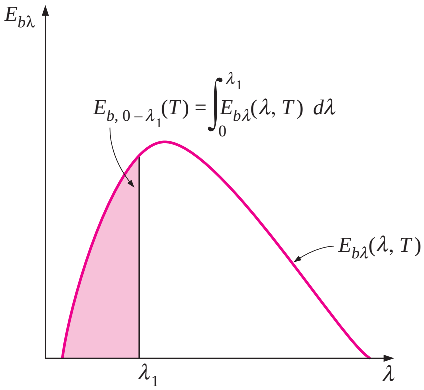


FIGURE 21–13
On an $E_{b\lambda}$–λ chart, the area under the curve to the left of the $\lambda = \lambda_1$ line represents the radiation energy emitted by a blackbody in the wavelength range 0–λ_1 for the given temperature.

TABLE 21–2

Blackbody radiation functions f_λ

λT, μm · K	f_λ	λT, μm · K	f_λ
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

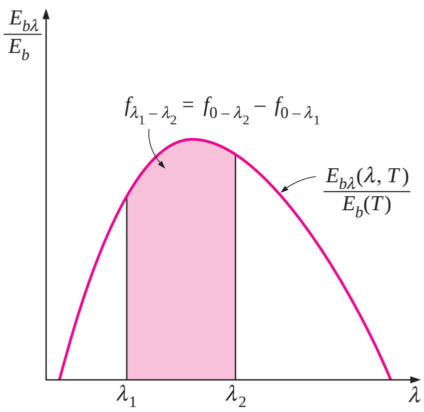


FIGURE 21–14
Graphical representation of the fraction of radiation emitted in the wavelength band from λ_1 to λ_2.

$$f_\lambda(T) = \frac{\int_0^\lambda E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4} \quad \textbf{(21–8)}$$

The function f_λ represents *the fraction of radiation emitted from a blackbody at temperature T in the wavelength band from $\lambda = 0$ to λ*. The values of f_λ are listed in Table 21–2 as a function of λT, where λ is in μm and T is in K.

The fraction of radiation energy emitted by a blackbody at temperature T over a finite wavelength band from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ is determined from (Fig. 21–14)

$$f_{\lambda_1-\lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T) \quad \textbf{(21–9)}$$

where $f_{\lambda_1}(T)$ and $f_{\lambda_2}(T)$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, respectively.

EXAMPLE 21–2 Emission of Radiation from a Lightbulb

The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

SOLUTION The temperature of the filament of an incandescent lightbulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a blackbody.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.4\ \mu$m to $\lambda_2 = 0.76\ \mu$m. Noting that $T = 2500$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 21–2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(2500\ \text{K}) = 1000\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_1} = 0.000321$$
$$\lambda_2 T = (0.76\ \mu\text{m})(2500\ \text{K}) = 1900\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_2} = 0.053035$$

That is, 0.03 percent of the radiation is emitted at wavelengths less than 0.4 μm and 5.3 percent at wavelengths less than 0.76 μm. Then the fraction of radiation emitted between these two wavelengths is (Fig. 21–15)

$$f_{\lambda_1-\lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = \mathbf{0.0527135}$$

Therefore, only about 5 percent of the radiation emitted by the filament of the lightbulb falls in the visible range. The remaining 95 percent of the radiation appears in the infrared region in the form of radiant heat or "invisible light," as it used to be called. This is certainly not a very efficient way of converting electrical energy to light and explains why fluorescent tubes are a wiser choice for lighting.

The wavelength at which the emission of radiation from the filament peaks is easily determined from Wien's displacement law to be

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \rightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2500\ \text{K}} = \mathbf{1.16\ \mu m}$$

Discussion Note that the radiation emitted from the filament peaks in the infrared region.

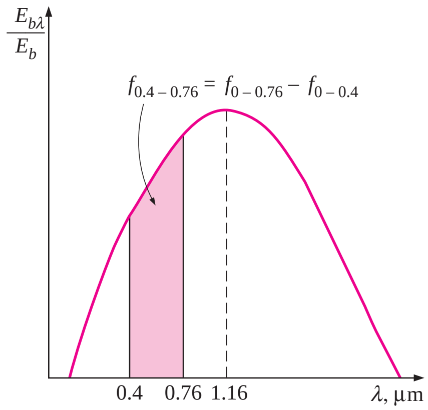


FIGURE 21–15
Graphical representation of the fraction of radiation emitted in the visible range in Example 21–2.

21–4 ▪ RADIATION INTENSITY

Radiation is emitted by all parts of a plane surface in all directions into the hemisphere above the surface, and the directional distribution of emitted (or incident) radiation is usually not uniform. Therefore, we need a quantity that describes the magnitude of radiation emitted (or incident) in a specified direction in space. This quantity is *radiation intensity,* denoted by I. Before we can describe a directional quantity, we need to specify direction in space. The direction of radiation passing through a point is best described in spherical coordinates in terms of the zenith angle θ and the azimuth angle ϕ, as shown in

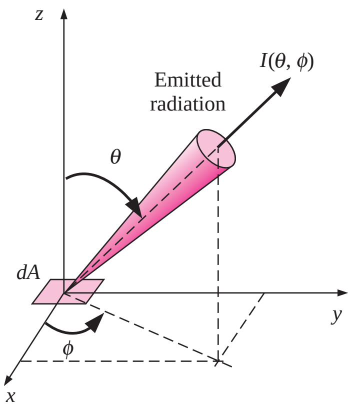
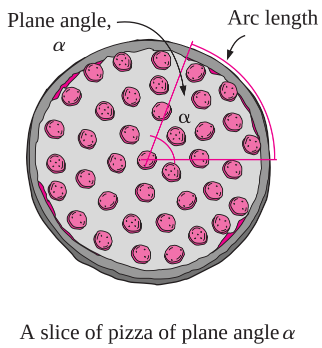


FIGURE 21–16
Radiation intensity is used to describe the variation of radiation energy with direction.



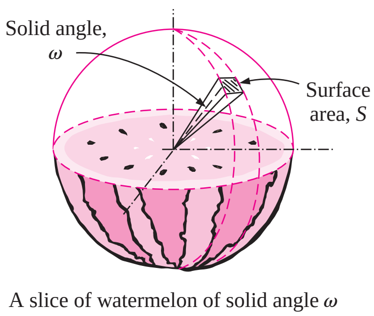


FIGURE 21–17
Describing the size of a slice of pizza by a plane angle, and the size of a watermelon slice by a solid angle.

Fig. 21–16. Radiation intensity is used to describe how the emitted radiation varies with the zenith and azimuth angles.

If all surfaces emitted radiation uniformly in all directions, the *emissive power* would be sufficient to quantify radiation, and we would not need to deal with intensity. The radiation emitted by a blackbody per unit normal area is the same in all directions, and thus there is no directional dependence. But this is not the case for real surfaces. Before we define intensity, we need to quantify the size of an opening in space.

Solid Angle

Let us try to quantify the size of a slice of pizza. One way of doing that is to specify the arc length of the outer edge of the slice, and to form the slice by connecting the endpoints of the arc to the center. A more general approach is to specify the angle of the slice at the center, as shown in Fig. 21–17. An angle of 90° (or $\pi/2$ radians), for example, always represents a quarter pizza, no matter what the radius is. For a circle of unit radius, the length of an arc is equivalent in magnitude to the *plane angle* it subtends (both are 2π for a complete circle of radius $r = 1$).

Now consider a watermelon, and let us attempt to quantify the size of a slice. Again we can do it by specifying the outer surface area of the slice (the green part), or by working with angles for generality. Connecting all points at the edges of the slice to the center in this case will form a three-dimensional body (like a cone whose tip is at the center), and thus the angle at the center in this case is properly called the **solid angle.** The solid angle is denoted by ω, and its unit is the *steradian* (sr). In analogy to plane angle, we can say that *the area of a surface on a sphere of unit radius is equivalent in magnitude to the solid angle it subtends* (both are 4π for a sphere of radius $r = 1$).

This can be shown easily by considering a differential surface area on a sphere $dS = r^2 \sin\theta\, d\theta\, d\phi$, as shown in Fig. 21–18, and integrating it from $\theta = 0$ to $\theta = \pi$, and from $\phi = 0$ to $\phi = 2\pi$. We get

$$S = \int_{\text{sphere}} dS = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 \sin\theta\, d\theta d\phi = 2\pi r^2 \int_{\theta=0}^{\pi} \sin\theta\, d\theta = 4\pi r^2 \quad \textbf{(21–10)}$$

which is the formula for the area of a sphere. For $r = 1$ it reduces to $S = 4\pi$, and thus the solid angle associated with a sphere is $\omega = 4\pi$ sr. For a hemisphere, which is more relevant to radiation emitted or received by a surface, it is $\omega = 2\pi$ sr.

The differential solid angle $d\omega$ subtended by a differential area dS on a sphere of radius r can be expressed as

$$d\omega = \frac{dS}{r^2} = \sin\theta\, d\theta\, d\phi \quad \textbf{(21–11)}$$

Note that the area dS is normal to the direction of viewing since dS is viewed from the center of the sphere. In general, the differential solid angle $d\omega$ subtended by a differential surface area dA when viewed from a point at a distance r from dA is expressed as

$$d\omega = \frac{dA_n}{r^2} = \frac{dA \cos\alpha}{r^2} \quad \textbf{(21–12)}$$

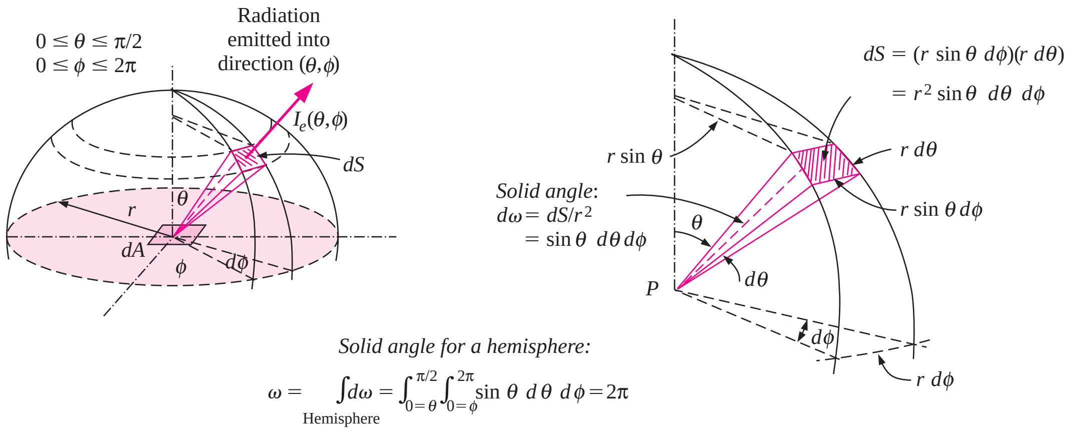


FIGURE 21–18
The emission of radiation from a differential surface element into the surrounding hemispherical space through a differential solid angle.

where α is the angle between the normal of the surface and the direction of viewing, and thus $dA_n = dA \cos \alpha$ is the normal (or projected) area to the direction of viewing.

Small surfaces viewed from relatively large distances can approximately be treated as differential areas in solid angle calculations. For example, the solid angle subtended by a 5 cm^2 plane surface when viewed from a point O at a distance of 80 cm along the normal of the surface is

$$\omega \cong \frac{A_n}{r^2} = \frac{5 \text{ cm}^2}{(80 \text{ cm})^2} = 7.81 \times 10^{-4} \text{ sr}$$

If the surface is tilted so that the normal of the surface makes an angle of $\alpha = 60°$ with the line connecting point O to the center of the surface, the projected area would be $dA_n = dA \cos \alpha = (5 \text{ cm}^2)\cos 60° = 2.5 \text{ cm}^2$, and the solid angle in this case would be half of the value just determined.

Intensity of Emitted Radiation

Consider the emission of radiation by a differential area element dA of a surface, as shown in Fig. 21–18. Radiation is emitted in all directions into the hemispherical space, and the radiation streaming though the surface area dS is proportional to the solid angle $d\omega$ subtended by dS. It is also proportional to the radiating area dA as seen by an observer on dS, which varies from a maximum of dA when dS is at the top directly above dA ($\theta = 0°$) to a minimum of zero when dS is at the bottom ($\theta = 90°$). Therefore, the effective area of dA for emission in the direction of θ is the projection of dA on a plane normal to θ, which is $dA \cos \theta$. Radiation intensity in a given direction is based on a unit area normal to that direction to provide a common basis for the comparison of radiation emitted in different directions.

The **radiation intensity** for emitted radiation $I_e(\theta, \phi)$ is defined as *the rate at which radiation energy $d\dot{Q}_e$ is emitted in the (θ, ϕ) direction per unit area normal to this direction and per unit solid angle about this direction.* That is,

$$I_e(\theta, \phi) = \frac{d\dot{Q}_e}{dA \cos \theta \cdot d\omega} = \frac{d\dot{Q}_e}{dA \cos \theta \sin \theta \, d\theta \, d\phi} \qquad (\text{W/m}^2 \cdot \text{sr}) \qquad \textbf{(21–13)}$$

The *radiation flux* for emitted radiation is the **emissive power** E (the rate at which radiation energy is emitted per unit area of the emitting surface), which can be expressed in differential form as

$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \qquad \textbf{(21–14)}$$

Noting that the hemisphere above the surface will intercept all the radiation rays emitted by the surface, the emissive power from the surface into the hemisphere surrounding it can be determined by integration as

$$E = \int_{\text{hemisphere}} dE = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_e(\theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \qquad (\text{W/m}^2) \qquad \textbf{(21–15)}$$

The intensity of radiation emitted by a surface, in general, varies with direction (especially with the zenith angle θ). But many surfaces in practice can be approximated as being diffuse. For a *diffusely emitting* surface, the intensity of the emitted radiation is independent of direction and thus I_e = constant. Noting that $\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi = \pi$, the emissive power relation in Eq. 21–15 reduces in this case to

Diffusely emitting surface: $$E = \pi I_e \qquad (\text{W/m}^2) \qquad \textbf{(21–16)}$$

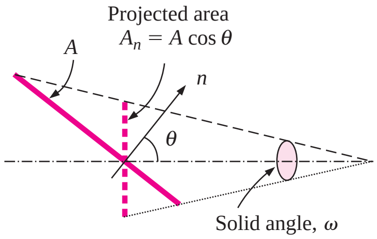


FIGURE 21–19
Radiation intensity is based on projected area, and thus the calculation of radiation emission from a surface involves the projection of the surface.

Note that the factor in Eq. 21–16 is π. You might have expected it to be 2π since intensity is radiation energy per unit solid angle, and the solid angle associated with a hemisphere is 2π. The reason for the factor being π is that the emissive power is based on the *actual* surface area whereas the intensity is based on the *projected* area (and thus the factor $\cos\theta$ that accompanies it), as shown in Fig. 21–19.

For a *blackbody,* which is a diffuse emitter, Eq. 21–16 can be expressed as

Blackbody: $$E_b = \pi I_b \qquad \textbf{(21–17)}$$

where $E_b = \sigma T^4$ is the blackbody emissive power. Therefore, the intensity of the radiation emitted by a blackbody at absolute temperature T is

Blackbody: $$I_b(T) = \frac{E_b(T)}{\pi} = \frac{\sigma T^4}{\pi} \qquad (\text{W/m}^2 \cdot \text{sr}) \qquad \textbf{(21–18)}$$

Incident Radiation

All surfaces emit radiation, but they also receive radiation emitted or reflected by other surfaces. The intensity of incident radiation $I_i(\theta, \phi)$ is defined as *the rate at which radiation energy dG is incident from the* (θ, ϕ) *direction per unit area of the receiving surface normal to this direction and per unit solid angle about this direction* (Fig. 21–20). Here θ is the angle between the direction of incident radiation and the normal of the surface.

The radiation flux incident on a surface from *all directions* is called **irradiation** G, and is expressed as

$$G = \int_{\text{hemisphere}} dG = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_i(\theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \qquad (\text{W/m}^2) \qquad \textbf{(21–19)}$$

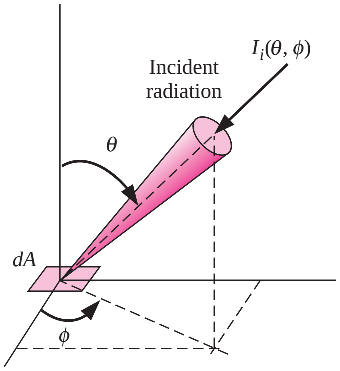


FIGURE 21–20
Radiation incident on a surface in the direction (θ, ϕ).

Therefore irradiation represents the rate at which radiation energy is incident on a surface per unit area of the surface. When the incident radiation is diffuse and thus I_i = constant, Eq. 21–19 reduces to

Diffusely incident radiation: $$G = \pi I_i \qquad (\text{W/m}^2) \tag{21–20}$$

Again note that irradiation is based on the *actual* surface area (and thus the factor cos θ), whereas the intensity of incident radiation is based on the *projected* area.

Radiosity

Surfaces emit radiation as well as reflecting it, and thus the radiation leaving a surface consists of emitted and reflected components, as shown in Fig. 21–21. The calculation of radiation heat transfer between surfaces involves the *total* radiation energy streaming away from a surface, with no regard for its origin. Thus, we need to define a quantity that represents *the rate at which radiation energy leaves a unit area of a surface in all directions.* This quantity is called the **radiosity** J, and is expressed as

$$J = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{e+r}(\theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \qquad (\text{W/m}^2) \tag{21–21}$$

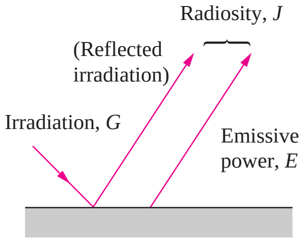


FIGURE 21–21
The three kinds of radiation flux (in W/m^2): emissive power, irradiation, and radiosity.

where I_{e+r} is the sum of the emitted and reflected intensities. For a surface that is both a diffuse emitter and a diffuse reflector, I_{e+r} = constant, and the radiosity relation reduces to

Diffuse emitter and reflector: $$J = \pi I_{e+r} \qquad (\text{W/m}^2) \tag{21–22}$$

For a blackbody, radiosity J is equivalent to the emissive power E_b since a blackbody absorbs the entire radiation incident on it and there is no reflected component in radiosity.

Spectral Quantities

So far we considered *total* radiation quantities (quantities integrated over all wavelengths), and made no reference to wavelength dependence. This lumped approach is adequate for many radiation problems encountered in practice. But sometimes it is necessary to consider the variation of radiation with wavelength as well as direction, and to express quantities at a certain wavelength λ or per unit wavelength interval about λ. Such quantities are referred to as *spectral* quantities to draw attention to wavelength dependence. The modifier "spectral" is used to indicate "at a given wavelength."

The *spectral radiation intensity* $I_\lambda(\lambda, \theta, \phi)$, for example, is simply the total radiation intensity $I(\theta, \phi)$ per unit wavelength interval about λ. The **spectral intensity** for emitted radiation $I_{\lambda, e}(\lambda, \theta, \phi)$ can be defined as *the rate at which radiation energy $d\dot{Q}_e$ is emitted at the wavelength λ in the (θ, ϕ) direction per unit area normal to this direction, per unit solid angle about this direction,* and it can be expressed as

$$I_{\lambda, e}(\lambda, \theta, \phi) = \frac{d\dot{Q}_e}{dA \cos\theta \cdot d\omega \cdot d\lambda} \qquad (\text{W/m}^2 \cdot \text{sr} \cdot \mu\text{m}) \tag{21–23}$$

Then the *spectral emissive power* becomes

$$E_\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \qquad (\text{W/m}^2) \tag{21–24}$$

Similar relations can be obtained for spectral irradiation G_λ, and spectral radiosity J_λ by replacing $I_{\lambda,e}$ in this equation by $I_{\lambda,i}$ and $I_{\lambda,e+r}$, respectively.

When the variation of spectral radiation intensity I_λ with wavelength λ is known, the total radiation intensity I for emitted, incident, and emitted + reflected radiation can be determined by integration over the entire wavelength spectrum as (Fig. 21–22)

$$I_e = \int_0^\infty I_{\lambda,e} \, d\lambda, \qquad I_i = \int_0^\infty I_{\lambda,i} \, d\lambda, \qquad \text{and} \qquad I_{e+r} = \int_0^\infty I_{\lambda,e+r} \, d\lambda \tag{21–25}$$

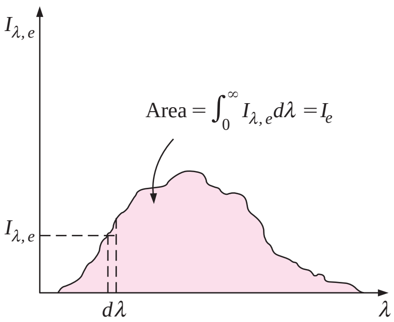


FIGURE 21–22
Integration of a "spectral" quantity for all wavelengths gives the "total" quantity.

These intensities can then be used in Eqs. 21–15, 21–19, and 21–21 to determine the emissive power E, irradiation G, and radiosity J, respectively.

Similarly, when the variations of spectral radiation fluxes E_λ, G_λ, and J_λ with wavelength λ are known, the total radiation fluxes can be determined by integration over the entire wavelength spectrum as

$$E = \int_0^\infty E_\lambda \, d\lambda, \qquad G = \int_0^\infty G_\lambda \, d\lambda, \quad \text{and} \qquad J = \int_0^\infty J_\lambda \, d\lambda \tag{21–26}$$

When the surfaces and the incident radiation are *diffuse*, the spectral radiation fluxes are related to spectral intensities as

$$E_\lambda = \pi I_{\lambda,e}, \qquad G_\lambda = \pi I_{\lambda,i}, \qquad \text{and} \qquad J_\lambda = \pi I_{\lambda,e+r} \tag{21–27}$$

Note that the relations for spectral and total radiation quantities are of the same form.

The spectral intensity of radiation emitted by a blackbody at an absolute temperature T at a wavelength λ has been determined by Max Planck, and is expressed as

$$I_{b\lambda}(\lambda, T) = \frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT) - 1]} \qquad (\text{W/m}^2 \cdot \text{sr} \cdot \mu\text{m}) \tag{21–28}$$

where $h = 6.6256 \times 10^{-34}$ J · s is the Planck constant, $k = 1.38065 \times 10^{-23}$ J/K is the Boltzmann constant, and $c_0 = 2.9979 \times 10^8$ m/s is the speed of light in a vacuum. Then the spectral blackbody emissive power is, from Eq. 21–27,

$$E_{b\lambda}(\lambda, T) = \pi I_{b\lambda}(\lambda, T) \tag{21–29}$$

A simplified relation for $E_{b\lambda}$ is given by Eq. 21–4.

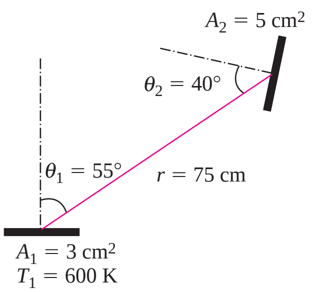


FIGURE 21–23
Schematic for Example 21–3.

EXAMPLE 21–3 **Radiation Incident on a Small Surface**

A small surface of area $A_1 = 3$ cm² emits radiation as a blackbody at $T_1 = 600$ K. Part of the radiation emitted by A_1 strikes another small surface of area $A_2 = 5$ cm² oriented as shown in Fig. 21–23. Determine the solid angle subtended by A_2 when viewed from A_1, and the rate at which radiation emitted by A_1 strikes A_2.

SOLUTION A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from Eq. 21–12 to be

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(5 \text{ cm}^2)\cos 40°}{(75 \text{ cm})^2} = \mathbf{6.81 \times 10^{-4} \text{ sr}}$$

since the normal of A_2 makes 40° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle ω_{2-1}. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(600 \text{ K})^4}{\pi} = 2339 \text{ W/m}^2 \cdot \text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1 \cos\theta_1)\omega_{2-1} \\ &= (2339 \text{ W/m}^2 \cdot \text{sr})(3 \times 10^{-4} \cos 55° \text{ m}^2)(6.81 \times 10^{-4} \text{ sr}) \\ &= \mathbf{2.74 \times 10^{-4} \text{ W}}\end{aligned}$$

Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.74×10^{-4} W.

Discussion The total rate of radiation emission from surface A_1 is $\dot{Q}_e = A_1\sigma T_1^4 = 2.204$ W. Therefore, the fraction of emitted radiation that strikes A_2 is $2.74 \times 10^{-4}/2.204 = 0.00012$ (or 0.012 percent). Noting that the solid angle associated with a hemisphere is 2π, the fraction of the solid angle subtended by A_2 is $6.81 \times 10^{-4}/(2\pi) = 0.000108$ (or 0.0108 percent), which is 0.9 times the fraction of emitted radiation. Therefore, the fraction of the solid angle a surface occupies does not represent the fraction of radiation energy the surface will receive even when the intensity of emitted radiation is constant. This is because radiation energy emitted by a surface in a given direction is proportional to the *projected area* of the surface in that direction, and reduces from a maximum at $\theta = 0°$ (the direction normal to surface) to zero at $\theta = 90°$ (the direction parallel to surface).

21–5 ▪ RADIATIVE PROPERTIES

Most materials encountered in practice, such as metals, wood, and bricks, are *opaque* to thermal radiation, and radiation is considered to be a *surface* *phenomenon* for such materials. That is, thermal radiation is emitted or absorbed within the first few microns of the surface, and thus we speak of radiative properties of *surfaces* for opaque materials.

Some other materials, such as glass and water, allow visible radiation to penetrate to considerable depths before any significant absorption takes place. Radiation through such *semitransparent* materials obviously cannot be considered to be a surface phenomenon since the entire volume of the material interacts with radiation. On the other hand, both glass and water are practically opaque to infrared radiation. Therefore, materials can exhibit different behavior at different wavelengths, and the dependence on wavelength is an important consideration in the study of radiative properties such as emissivity, absorptivity, reflectivity, and transmissivity of materials.

In the preceding section, we defined a *blackbody* as a perfect emitter and absorber of radiation and said that no body can emit more radiation than a blackbody at the same temperature. Therefore, a blackbody can serve as a convenient *reference* in describing the emission and absorption characteristics of real surfaces.

Emissivity

The **emissivity** of a surface represents *the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature.* The emissivity of a surface is denoted by ε, and it varies between zero and one, $0 \leq \varepsilon \leq 1$. Emissivity is a measure of how closely a surface approximates a blackbody, for which $\varepsilon = 1$.

The emissivity of a real surface is not a constant. Rather, it varies with the *temperature* of the surface as well as the *wavelength* and the *direction* of the emitted radiation. Therefore, different emissivities can be defined for a surface, depending on the effects considered. The most elemental emissivity of a surface at a given temperature is the **spectral directional emissivity,** which is defined as the ratio of the intensity of radiation emitted by the surface at a specified wavelength in a specified direction to the intensity of radiation emitted by a blackbody at the same temperature at the same wavelength. That is,

$$\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda,e}(\lambda, \theta, \phi, T)}{I_{b\lambda}(\lambda, T)} \tag{21–30}$$

where the subscripts λ and θ are used to designate *spectral* and *directional* quantities, respectively. Note that blackbody radiation intensity is independent of direction, and thus it has no functional dependence on θ and ϕ.

The **total directional emissivity** is defined in a like manner by using total intensities (intensities integrated over all wavelengths) as

$$\varepsilon_{\theta}(\theta, \phi, T) = \frac{I_e(\theta, \phi, T)}{I_b(T)} \tag{21–31}$$

In practice, it is usually more convenient to work with radiation properties averaged over all directions, called *hemispherical properties.* Noting that the integral of the rate of radiation energy emitted at a specified wavelength per unit surface area over the entire hemisphere is *spectral emissive power,* the **spectral hemispherical emissivity** can be expressed as

$$\varepsilon_\lambda(\lambda, T) = \frac{E_\lambda(\lambda, T)}{E_{b\lambda}(\lambda, T)} \quad \textbf{(21–32)}$$

Note that the emissivity of a surface at a given wavelength can be different at different temperatures since the spectral distribution of emitted radiation (and thus the amount of radiation emitted at a given wavelength) changes with temperature.

Finally, the **total hemispherical emissivity** is defined in terms of the radiation energy emitted over all wavelengths in all directions as

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \quad \textbf{(21–33)}$$

Therefore, the total hemispherical emissivity (or simply the "average emissivity") of a surface at a given temperature represents the ratio of the total radiation energy emitted by the surface to the radiation emitted by a blackbody of the same surface area at the same temperature.

Noting from Eqs. 21–26 and 21–32 that $E = \int_0^\infty E_\lambda d\lambda$ and $E_\lambda(\lambda, T) = \varepsilon_\lambda(\lambda, T)E_{b\lambda}(\lambda, T)$, and the total hemispherical emissivity can also be expressed as

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T)E_{b\lambda}(\lambda, T)d\lambda}{\sigma T^4} \quad \textbf{(21–34)}$$

since $E_b(T) = \sigma T^4$. To perform this integration, we need to know the variation of spectral emissivity with wavelength at the specified temperature. The integrand is usually a complicated function, and the integration has to be performed numerically. However, the integration can be performed quite easily by dividing the spectrum into a sufficient number of *wavelength bands* and assuming the emissivity to remain constant over each band; that is, by expressing the function $\varepsilon_\lambda(\lambda, T)$ as a step function. This simplification offers great convenience for little sacrifice of accuracy, since it allows us to transform the integration into a summation in terms of blackbody emission functions.

As an example, consider the emissivity function plotted in Fig. 21–24. It seems like this function can be approximated reasonably well by a step function of the form

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = \text{constant}, & 0 \le \lambda < \lambda_1 \\ \varepsilon_2 = \text{constant}, & \lambda_1 \le \lambda < \lambda_2 \\ \varepsilon_3 = \text{constant}, & \lambda_2 \le \lambda < \infty \end{cases} \quad \textbf{(21–35)}$$

Then the average emissivity can be determined from Eq. 21–34 by breaking the integral into three parts and utilizing the definition of the blackbody radiation function as

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{E_b}$$
$$= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T) \quad \textbf{(21–36)}$$

Radiation is a complex phenomenon as it is, and the consideration of the wavelength and direction dependence of properties, assuming sufficient data

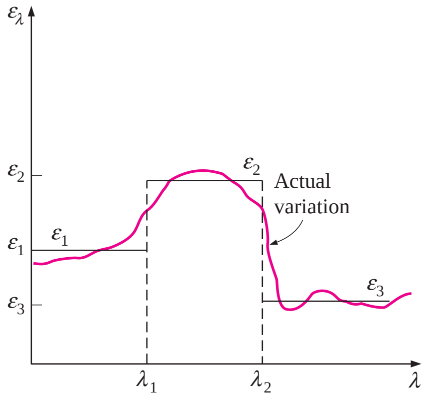


FIGURE 21–24
Approximating the actual variation of emissivity with wavelength by a step function.

Real surface:
$\varepsilon_\theta \neq$ constant
$\varepsilon_\lambda \neq$ constant

Diffuse surface:
$\varepsilon_\theta =$ constant

Gray surface:
$\varepsilon_\lambda =$ constant

Diffuse, gray surface:
$\varepsilon = \varepsilon_\lambda = \varepsilon_\theta =$ constant

FIGURE 21–25
The effect of diffuse and gray approximations on the emissivity of a surface.

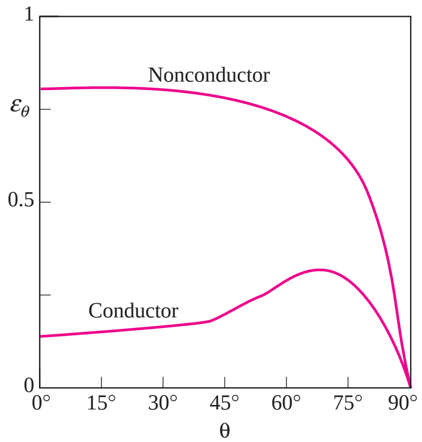


FIGURE 21–26
Typical variations of emissivity with direction for electrical conductors and nonconductors.

exist, makes it even more complicated. Therefore, the *gray* and *diffuse* approximations are often utilized in radiation calculations. A surface is said to be *diffuse* if its properties are *independent of direction*, and *gray* if its properties are *independent of wavelength*. Therefore, the emissivity of a gray, diffuse surface is simply the total hemispherical emissivity of that surface because of independence of direction and wavelength (Fig. 21–25).

A few comments about the validity of the diffuse approximation are in order. Although real surfaces do not emit radiation in a perfectly diffuse manner as a blackbody does, they often come close. The variation of emissivity with direction for both electrical conductors and nonconductors is given in Fig. 21–26. Here θ is the angle measured from the normal of the surface, and thus $\theta = 0$ for radiation emitted in a direction normal to the surface. Note that ε_θ remains nearly constant for about $\theta < 40°$ for conductors such as metals and for $\theta < 70°$ for nonconductors such as plastics. Therefore, the directional emissivity of a surface in the normal direction is representative of the hemispherical emissivity of the surface. In radiation analysis, it is common practice to assume the surfaces to be diffuse emitters with an emissivity equal to the value in the normal ($\theta = 0$) direction.

The effect of the gray approximation on emissivity and emissive power of a real surface is illustrated in Fig. 21–27. Note that the radiation emission from a real surface, in general, differs from the Planck distribution, and the emission curve may have several peaks and valleys. A gray surface should emit as much radiation as the real surface it represents at the same temperature. Therefore, the areas under the emission curves of the real and gray surfaces must be equal.

The emissivities of common materials are listed in Table A–30 in the appendix, and the variation of emissivity with wavelength and temperature is illustrated in Fig. 21–28. Typical ranges of emissivity of various materials are given in Fig. 21–29. Note that metals generally have low emissivities, as low as 0.02 for polished surfaces, and nonmetals such as ceramics and organic materials have high ones. The emissivity of metals increases with temperature. Also, oxidation causes significant increases in the emissivity of metals. Heavily oxidized metals can have emissivities comparable to those of nonmetals.

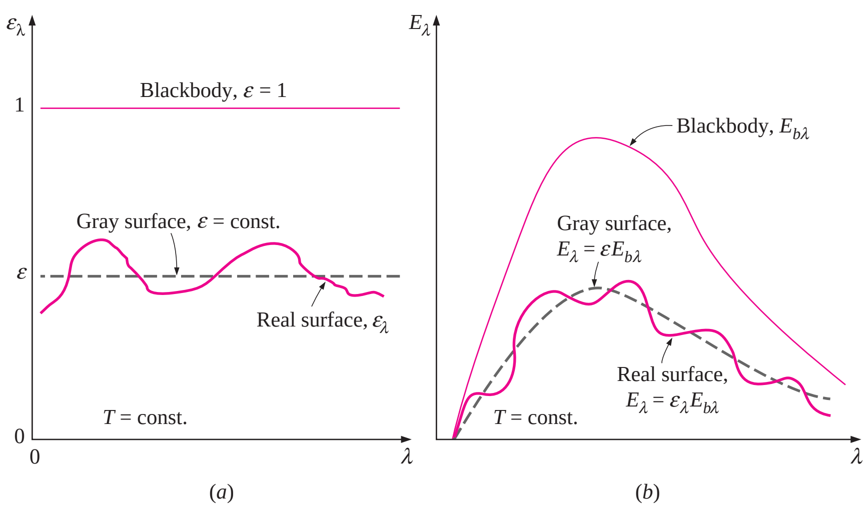


FIGURE 21–27
Comparison of the emissivity (*a*) and emissive power (*b*) of a real surface with those of a gray surface and a blackbody at the same temperature.

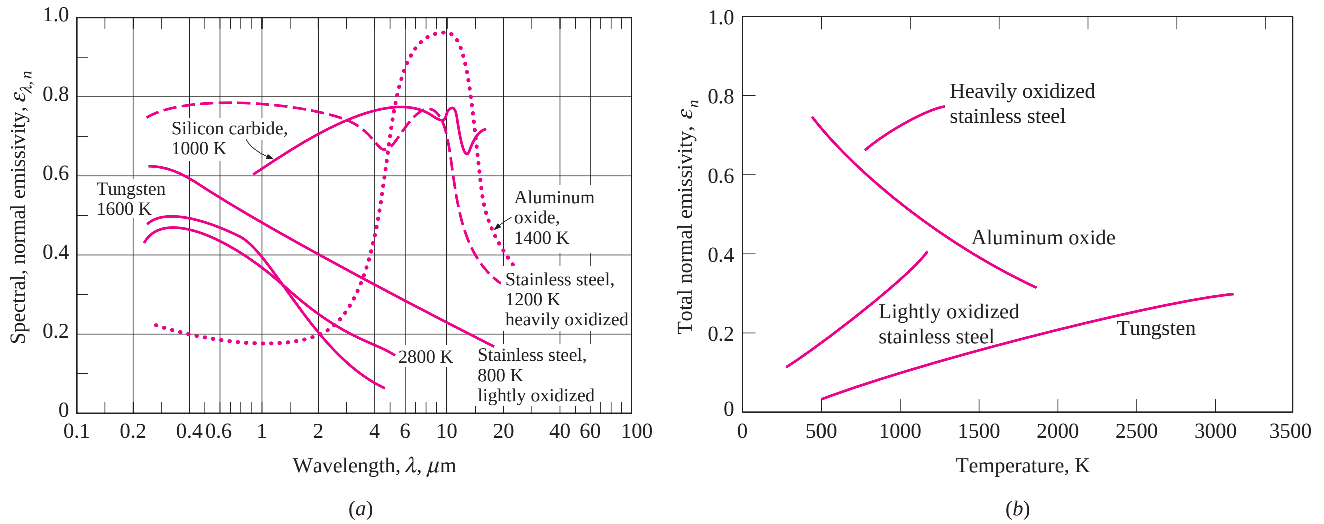


FIGURE 21–28
The variation of normal emissivity with (*a*) wavelength and (*b*) temperature for various materials.

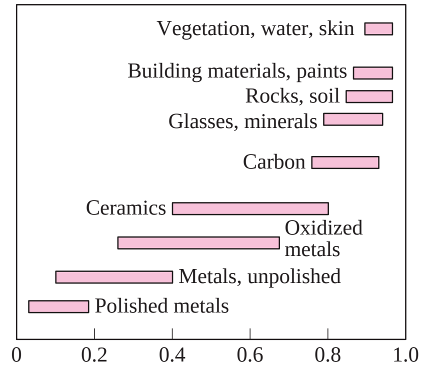


FIGURE 21–29
Typical ranges of emissivity for various materials.

Care should be exercised in the use and interpretation of radiation property data reported in the literature, since the properties strongly depend on the surface conditions such as oxidation, roughness, type of finish, and cleanliness. Consequently, there is considerable discrepancy and uncertainty in the reported values. This uncertainty is largely due to the difficulty in characterizing and describing the surface conditions precisely.

EXAMPLE 21–4 Emissivity of a Surface and Emissive Power

The spectral emissivity function of an opaque surface at 800 K is approximated as (Fig. 21–30)

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.3, & 0 \le \lambda < 3\ \mu\text{m} \\ \varepsilon_2 = 0.8, & 3\ \mu\text{m} \le \lambda < 7\ \mu\text{m} \\ \varepsilon_3 = 0.1, & 7\ \mu\text{m} \le \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and its emissive power.

SOLUTION The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The variation of the emissivity of the surface with wavelength is given as a step function. Therefore, the average emissivity of the surface can be determined from Eq. 21–34 by breaking the integral into three parts,

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{\sigma T^4}$$
$$= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$
$$= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

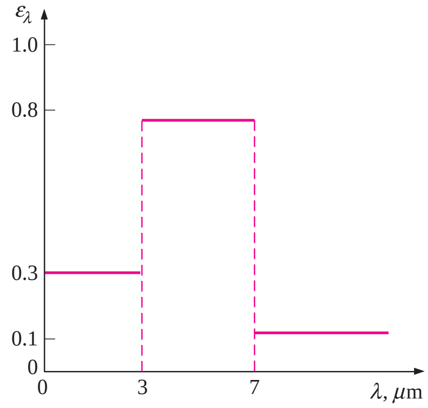


FIGURE 21–30
The spectral emissivity of the surface considered in Example 21–4.

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 21–2 to be

$$\lambda_1 T = (3\ \mu\text{m})(800\ \text{K}) = 2400\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.140256$$
$$\lambda_2 T = (7\ \mu\text{m})(800\ \text{K}) = 5600\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_2} = 0.701046$$

Note that $f_{0-\lambda_1} = f_{\lambda_1} - f_0 = f_{\lambda_1}$, since $f_0 = 0$, and $f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} = 1 - f_{\lambda_2}$, since $f_\infty = 1$. Substituting,

$$\varepsilon = 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)$$
$$= \mathbf{0.521}$$

That is, the surface will emit as much radiation energy at 800 K as a gray surface having a constant emissivity of $\varepsilon = 0.521$. The emissive power of the surface is

$$E = \varepsilon\sigma T^4 = 0.521(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(800\ \text{K})^4 = \mathbf{12{,}100\ W/m^2}$$

Discussion Note that the surface emits 12.1 kJ of radiation energy per second per m^2 area of the surface.

Absorptivity, Reflectivity, and Transmissivity

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that every body, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths. Recall that radiation flux *incident on a surface* is called **irradiation** and is denoted by G.

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Fig. 21–31. *The fraction of irradiation absorbed by the surface* is called the **absorptivity** α, *the fraction reflected by the surface* is called the **reflectivity** ρ, and *the fraction transmitted* is called the **transmissivity** τ. That is,

Absorptivity: $$\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{abs}}{G}, \qquad 0 \le \alpha \le 1 \qquad \textbf{(21–37)}$$

Reflectivity: $$\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{ref}}{G}, \qquad 0 \le \rho \le 1 \qquad \textbf{(21–38)}$$

Transmissivity: $$\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{tr}}{G}, \qquad 0 \le \tau \le 1 \qquad \textbf{(21–39)}$$

where G is the radiation energy incident on the surface, and G_{abs}, G_{ref}, and G_{tr} are the absorbed, reflected, and transmitted portions of it, respectively. The first law of thermodynamics requires that the sum of the absorbed, reflected, and transmitted radiation energy be equal to the incident radiation. That is,

$$G_{abs} + G_{ref} + G_{tr} = G \qquad \textbf{(21–40)}$$

Dividing each term of this relation by G yields

$$\alpha + \rho + \tau = 1 \qquad \textbf{(21–41)}$$

For opaque surfaces, $\tau = 0$, and thus

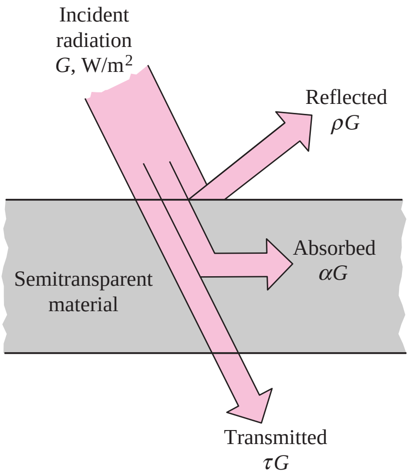


FIGURE 21–31
The absorption, reflection, and transmission of incident radiation by a semitransparent material.