

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 21–2 to be

$$\lambda_1 T = (3 \mu\text{m})(800 \text{ K}) = 2400 \mu\text{m} \cdot \text{K} \rightarrow f_{\lambda_1} = 0.140256$$

$$\lambda_2 T = (7 \mu\text{m})(800 \text{ K}) = 5600 \mu\text{m} \cdot \text{K} \rightarrow f_{\lambda_2} = 0.701046$$

Note that $f_{0-\lambda_1} = f_{\lambda_1} - f_0 = f_{\lambda_1}$, since $f_0 = 0$, and $f_{\lambda_2-\infty} = f_{\infty} - f_{\lambda_2} = 1 - f_{\lambda_2}$, since $f_{\infty} = 1$. Substituting,

$$\begin{aligned} \varepsilon &= 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046) \\ &= \mathbf{0.521} \end{aligned}$$

That is, the surface will emit as much radiation energy at 800 K as a gray surface having a constant emissivity of $\varepsilon = 0.521$. The emissive power of the surface is

$$E = \varepsilon \sigma T^4 = 0.521(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(800 \text{ K})^4 = \mathbf{12,100 \text{ W/m}^2}$$

Discussion Note that the surface emits 12.1 kJ of radiation energy per second per m^2 area of the surface.

Absorptivity, Reflectivity, and Transmissivity

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that every body, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths. Recall that radiation flux *incident on a surface* is called **irradiation** and is denoted by G .

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Fig. 21–31. *The fraction of irradiation absorbed by the surface* is called the **absorptivity** α , *the fraction reflected by the surface* is called the **reflectivity** ρ , and *the fraction transmitted* is called the **transmissivity** τ . That is,

$$\text{Absorptivity:} \quad \alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{\text{abs}}}{G}, \quad 0 \leq \alpha \leq 1 \quad (21-37)$$

$$\text{Reflectivity:} \quad \rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{\text{ref}}}{G}, \quad 0 \leq \rho \leq 1 \quad (21-38)$$

$$\text{Transmissivity:} \quad \tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{\text{tr}}}{G}, \quad 0 \leq \tau \leq 1 \quad (21-39)$$

where G is the radiation energy incident on the surface, and G_{abs} , G_{ref} , and G_{tr} are the absorbed, reflected, and transmitted portions of it, respectively. The first law of thermodynamics requires that the sum of the absorbed, reflected, and transmitted radiation energy be equal to the incident radiation. That is,

$$G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} = G \quad (21-40)$$

Dividing each term of this relation by G yields

$$\alpha + \rho + \tau = 1 \quad (21-41)$$

For opaque surfaces, $\tau = 0$, and thus

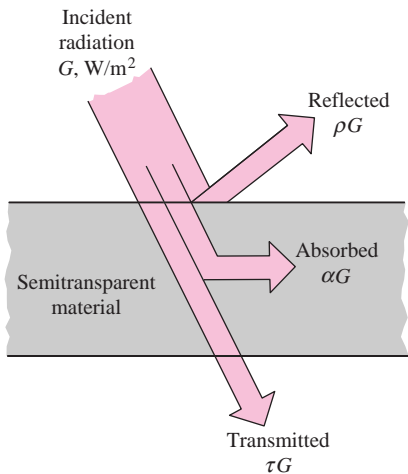
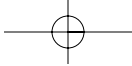


FIGURE 21–31

The absorption, reflection, and transmission of incident radiation by a semitransparent material.



$$\alpha + \rho = 1 \quad (21-42)$$

This is an important property relation since it enables us to determine both the absorptivity and reflectivity of an opaque surface by measuring either of these properties.

These definitions are for *total hemispherical* properties, since G represents the radiation flux incident on the surface from all directions over the hemispherical space and over all wavelengths. Thus, α , ρ , and τ are the *average* properties of a medium for all directions and all wavelengths. However, like emissivity, these properties can also be defined for a specific wavelength and/or direction. For example, the **spectral directional absorptivity** and **spectral directional reflectivity** of a surface are defined, respectively, as the absorbed and reflected fractions of the intensity of radiation incident at a specified wavelength in a specified direction as

$$\alpha_{\lambda, \theta}(\lambda, \theta, \phi) = \frac{I_{\lambda, \text{abs}}(\lambda, \theta, \phi)}{I_{\lambda, i}(\lambda, \theta, \phi)} \quad \text{and} \quad \rho_{\lambda, \theta}(\lambda, \theta, \phi) = \frac{I_{\lambda, \text{ref}}(\lambda, \theta, \phi)}{I_{\lambda, i}(\lambda, \theta, \phi)} \quad (21-43)$$

Likewise, the **spectral hemispherical absorptivity** and **spectral hemispherical reflectivity** of a surface are defined as

$$\alpha_{\lambda}(\lambda) = \frac{G_{\lambda, \text{abs}}(\lambda)}{G_{\lambda}(\lambda)} \quad \text{and} \quad \rho_{\lambda}(\lambda) = \frac{G_{\lambda, \text{ref}}(\lambda)}{G_{\lambda}(\lambda)} \quad (21-44)$$

where G_{λ} is the spectral irradiation (in $\text{W}/\text{m}^2 \cdot \mu\text{m}$) incident on the surface, and $G_{\lambda, \text{abs}}$ and $G_{\lambda, \text{ref}}$ are the reflected and absorbed portions of it, respectively.

Similar quantities can be defined for the transmissivity of semitransparent materials. For example, the **spectral hemispherical transmissivity** of a medium can be expressed as

$$\tau_{\lambda}(\lambda) = \frac{G_{\lambda, \text{tr}}(\lambda)}{G_{\lambda}(\lambda)} \quad (21-45)$$

The average absorptivity, reflectivity, and transmissivity of a surface can also be defined in terms of their spectral counterparts as

$$\alpha = \frac{\int_0^{\infty} \alpha_{\lambda} G_{\lambda} d\lambda}{\int_0^{\infty} G_{\lambda} d\lambda}, \quad \rho = \frac{\int_0^{\infty} \rho_{\lambda} G_{\lambda} d\lambda}{\int_0^{\infty} G_{\lambda} d\lambda}, \quad \tau = \frac{\int_0^{\infty} \tau_{\lambda} G_{\lambda} d\lambda}{\int_0^{\infty} G_{\lambda} d\lambda} \quad (21-46)$$

The reflectivity differs somewhat from the other properties in that it is *bidirectional* in nature. That is, the value of the reflectivity of a surface depends not only on the direction of the incident radiation but also the direction of reflection. Therefore, the reflected rays of a radiation beam incident on a real surface in a specified direction will form an irregular shape, as shown in Fig. 21–32. Such detailed reflectivity data do not exist for most surfaces, and even if they did, they would be of little value in radiation calculations since this would usually add more complication to the analysis than it is worth.

In practice, for simplicity, surfaces are assumed to reflect in a perfectly *specular* or *diffuse* manner. In **specular** (or *mirrorlike*) **reflection**, the angle of reflection equals the angle of incidence of the radiation beam. In **diffuse reflection**, radiation is reflected equally in all directions, as shown in Fig.

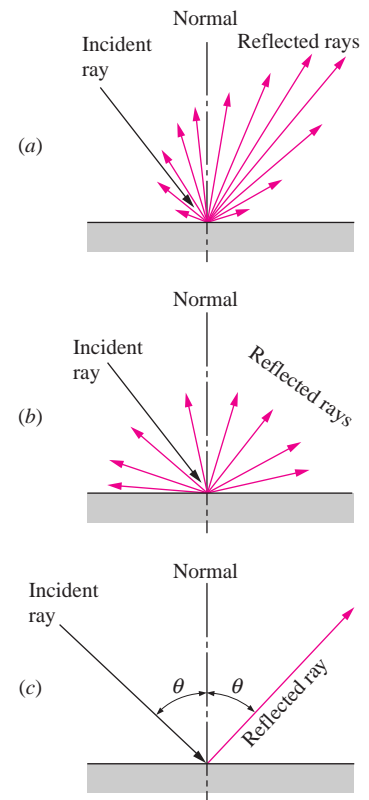
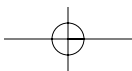


FIGURE 21–32

Different types of reflection from a surface: (a) actual or irregular, (b) diffuse, and (c) specular or mirrorlike.



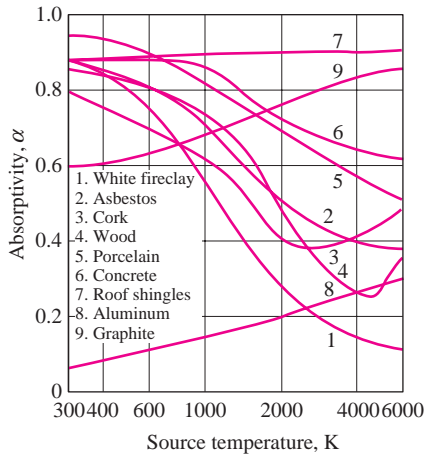
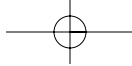


FIGURE 21-33 Variation of absorptivity with the temperature of the source of irradiation for various common materials at room temperature.

21-32. Reflection from smooth and polished surfaces approximates specular reflection, whereas reflection from rough surfaces approximates diffuse reflection. In radiation analysis, smoothness is defined relative to wavelength. A surface is said to be *smooth* if the height of the surface roughness is much smaller than the wavelength of the incident radiation.

Unlike emissivity, the absorptivity of a material is practically independent of surface temperature. However, the absorptivity depends strongly on the temperature of the source at which the incident radiation is originating. This is also evident from Fig. 21-33, which shows the absorptivities of various materials at room temperature as functions of the temperature of the radiation source. For example, the absorptivity of the concrete roof of a house is about 0.6 for solar radiation (source temperature: 5780 K) and 0.9 for radiation originating from the surrounding trees and buildings (source temperature: 300 K), as illustrated in Fig. 21-34.

Notice that the absorptivity of aluminum increases with the source temperature, a characteristic for metals, and the absorptivity of electric nonconductors, in general, decreases with temperature. This decrease is most pronounced for surfaces that appear white to the eye. For example, the absorptivity of a white painted surface is low for solar radiation, but it is rather high for infrared radiation.

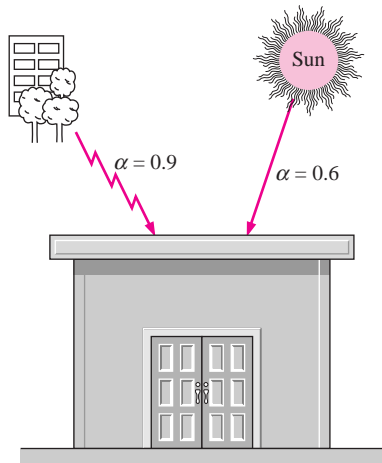


FIGURE 21-34 The absorptivity of a material may be quite different for radiation originating from sources at different temperatures.

Kirchhoff's Law

Consider a small body of surface area A_s , emissivity ϵ , and absorptivity α at temperature T contained in a large isothermal enclosure at the same temperature, as shown in Fig. 21-35. Recall that a large isothermal enclosure forms a blackbody cavity regardless of the radiative properties of the enclosure surface, and the body in the enclosure is too small to interfere with the blackbody nature of the cavity. Therefore, the radiation incident on any part of the surface of the small body is equal to the radiation emitted by a blackbody at temperature T . That is, $G = E_b(T) = \sigma T^4$, and the radiation absorbed by the small body per unit of its surface area is

$$G_{\text{abs}} = \alpha G = \alpha \sigma T^4$$

The radiation emitted by the small body is

$$E_{\text{emit}} = \epsilon \sigma T^4$$

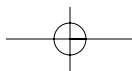
Considering that the small body is in thermal equilibrium with the enclosure, the net rate of heat transfer to the body must be zero. Therefore, the radiation emitted by the body must be equal to the radiation absorbed by it:

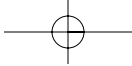
$$A_s \epsilon \sigma T^4 = A_s \alpha \sigma T^4$$

Thus, we conclude that

$$\epsilon(T) = \alpha(T) \tag{21-47}$$

That is, *the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature.* This relation, which greatly simplifies the radiation analysis, was first developed by Gustav Kirchhoff in 1860 and is now called **Kirchhoff's law**. Note that this relation is derived under the condition





that the surface temperature is equal to the temperature of the source of irradiation, and the reader is cautioned against using it when considerable difference (more than a few hundred degrees) exists between the surface temperature and the temperature of the source of irradiation.

The derivation above can also be repeated for radiation at a specified wavelength to obtain the *spectral* form of Kirchhoff's law:

$$\varepsilon_{\lambda}(T) = \alpha_{\lambda}(T) \quad (21-48)$$

This relation is valid when the irradiation or the emitted radiation is independent of direction. The form of Kirchhoff's law that involves no restrictions is the *spectral directional* form expressed as $\varepsilon_{\lambda, \theta}(T) = \alpha_{\lambda, \theta}(T)$. That is, the emissivity of a surface at a specified wavelength, direction, and temperature is always equal to its absorptivity at the same wavelength, direction, and temperature.

It is very tempting to use Kirchhoff's law in radiation analysis since the relation $\varepsilon = \alpha$ together with $\rho = 1 - \alpha$ enables us to determine all three properties of an opaque surface from a knowledge of only *one* property. Although Eq. 21-47 gives acceptable results in most cases, in practice, care should be exercised when there is considerable difference between the surface temperature and the temperature of the source of incident radiation.

The Greenhouse Effect

You have probably noticed that when you leave your car under direct sunlight on a sunny day, the interior of the car gets much warmer than the air outside, and you may have wondered why the car acts like a *heat trap*. The answer lies in the spectral transmissivity curve of the *glass*, which resembles an inverted U, as shown in Fig. 21-36. We observe from this figure that glass at thicknesses encountered in practice transmits over 90 percent of radiation in the visible range and is practically opaque (nontransparent) to radiation in the longer-wavelength infrared regions of the electromagnetic spectrum (roughly $\lambda > 3 \mu\text{m}$). Therefore, glass has a transparent window in the wavelength range $0.3 \mu\text{m} < \lambda < 3 \mu\text{m}$ in which over 90 percent of solar radiation is emitted. On the other hand, the entire radiation emitted by surfaces at room temperature falls in the infrared region. Consequently, glass allows the solar radiation to enter but does not allow the infrared radiation from the interior surfaces to escape. This causes a rise in the interior temperature as a result of the energy buildup in the car. This heating effect, which is due to the nongray characteristic of glass (or clear plastics), is known as the **greenhouse effect**, since it is utilized primarily in greenhouses (Fig. 21-37).

The greenhouse effect is also experienced on a larger scale on earth. The surface of the earth, which warms up during the day as a result of the absorption of solar energy, cools down at night by radiating its energy into deep space as infrared radiation. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth. Thus, there is concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

In *humid* places such as coastal areas, there is not a large change between the daytime and nighttime temperatures, because the humidity acts as a barrier

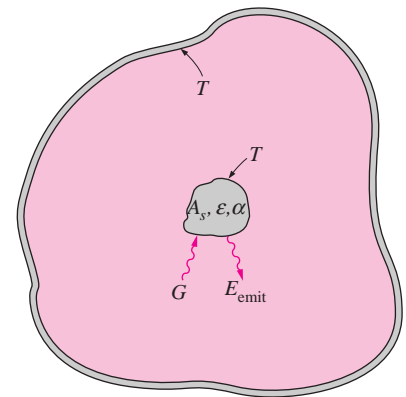


FIGURE 21-35

The small body contained in a large isothermal enclosure used in the development of Kirchhoff's law.

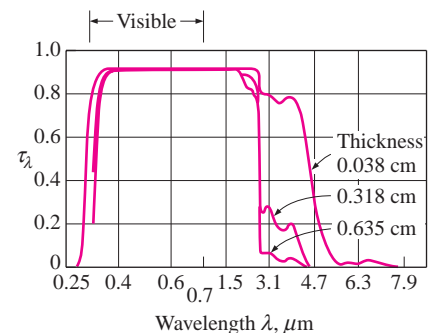


FIGURE 21-36

The spectral transmissivity of low-iron glass at room temperature for different thicknesses.

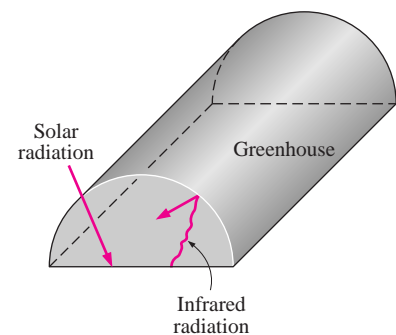
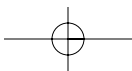


FIGURE 21-37

A greenhouse traps energy by allowing the solar radiation to come in but not allowing the infrared radiation to go out.



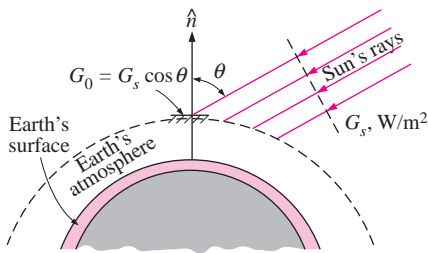
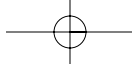


FIGURE 21-38 Solar radiation reaching the earth's atmosphere and the total solar irradiance.

on the path of the infrared radiation coming from the earth, and thus slows down the cooling process at night. In areas with clear skies such as deserts, there is a large swing between the daytime and nighttime temperatures because of the absence of such barriers for infrared radiation.

21-6 ■ ATMOSPHERIC AND SOLAR RADIATION

The sun is our primary source of energy. The energy coming off the sun, called *solar energy*, reaches us in the form of electromagnetic waves after experiencing considerable interactions with the atmosphere. The radiation energy emitted or reflected by the constituents of the atmosphere form the *atmospheric radiation*. Here we give an overview of the solar and atmospheric radiation because of their importance and relevance to daily life. Also, our familiarity with solar energy makes it an effective tool in developing a better understanding for some of the new concepts introduced earlier. Detailed treatment of this exciting subject can be found in numerous books devoted to this topic.

The *sun* is a nearly spherical body that has a diameter of $D \approx 1.39 \times 10^9$ m and a mass of $m \approx 2 \times 10^{30}$ kg and is located at a mean distance of $L = 1.50 \times 10^{11}$ m from the earth. It emits radiation energy continuously at a rate of $E_{\text{sun}} \approx 3.8 \times 10^{26}$ W. Less than a billionth of this energy (about 1.7×10^{17} W) strikes the earth, which is sufficient to keep the earth warm and to maintain life through the photosynthesis process. The energy of the sun is due to the continuous *fusion* reaction during which two hydrogen atoms fuse to form one atom of helium. Therefore, the sun is essentially a *nuclear reactor*, with temperatures as high as 40,000,000 K in its core region. The temperature drops to about 5800 K in the outer region of the sun, called the convective zone, as a result of the dissipation of this energy by radiation.

The solar energy reaching the earth's atmosphere is called the **total solar irradiance** G_s , whose value is

$$G_s = 1373 \text{ W/m}^2 \tag{21-49}$$

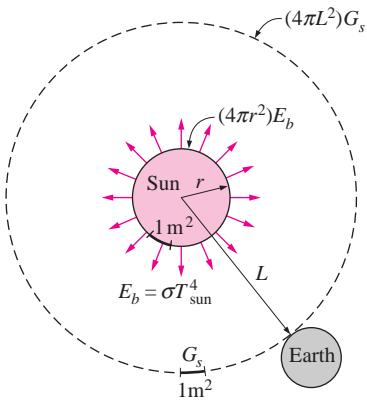


FIGURE 21-39 The total solar energy passing through concentric spheres remains constant, but the energy falling per unit area decreases with increasing radius.

The total solar irradiance (also called the **solar constant**) represents *the rate at which solar energy is incident on a surface normal to the sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun* (Fig. 21-38).

The value of the total solar irradiance can be used to estimate the effective surface temperature of the sun from the requirement that

$$(4\pi L^2)G_s = (4\pi r^2)\sigma T_{\text{sun}}^4 \tag{21-50}$$

where L is the mean distance between the sun's center and the earth and r is the radius of the sun. The left-hand side of this equation represents the total solar energy passing through a spherical surface whose radius is the mean earth-sun distance, and the right-hand side represents the total energy that leaves the sun's outer surface. The conservation of energy principle requires that these two quantities be equal to each other, since the solar energy experiences no attenuation (or enhancement) on its way through the vacuum (Fig. 21-39). The **effective surface temperature** of the sun is determined from Eq. 21-50 to be $T_{\text{sun}} = 5780$ K. That is, the sun can be treated as a

