**FIGURE 22-1**

Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, and this dependence on orientation is accounted for by the *view factor*.

## 22-1 ■ THE VIEW FACTOR

Radiation heat transfer between surfaces depends on the *orientation* of the surfaces relative to each other as well as their radiation properties and temperatures, as illustrated in Fig. 22-1. For example, a camper will make the most use of a campfire on a cold night by standing as close to the fire as possible and by blocking as much of the radiation coming from the fire by turning her front to the fire instead of her side. Likewise, a person will maximize the amount of solar radiation incident on him and take a sunbath by lying down on his back instead of standing up on his feet.

To account for the effects of orientation on radiation heat transfer between two surfaces, we define a new parameter called the *view factor*, which is a purely geometric quantity and is independent of the surface properties and temperature. It is also called the *shape factor*, *configuration factor*, and *angle factor*. The view factor based on the assumption that the surfaces are diffuse emitters and diffuse reflectors is called the *diffuse view factor*, and the view factor based on the assumption that the surfaces are diffuse emitters but specular reflectors is called the *specular view factor*. In this book, we will consider radiation exchange between diffuse surfaces only, and thus the term *view factor* will simply mean *diffuse view factor*.

The view factor from a surface  $i$  to a surface  $j$  is denoted by  $F_{i \rightarrow j}$  or just  $F_{ij}$ , and is defined as

$$F_{ij} = \text{the fraction of the radiation leaving surface } i \text{ that strikes surface } j \text{ directly}$$

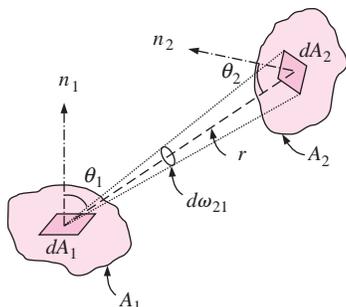
The notation  $F_{i \rightarrow j}$  is instructive for beginners, since it emphasizes that the view factor is for radiation that travels from surface  $i$  to surface  $j$ . However, this notation becomes rather awkward when it has to be used many times in a problem. In such cases, it is convenient to replace it by its *shorthand* version  $F_{ij}$ .

Therefore, the view factor  $F_{12}$  represents the fraction of radiation leaving surface 1 that strikes surface 2 directly, and  $F_{21}$  represents the fraction of the radiation leaving surface 2 that strikes surface 1 directly. Note that the radiation that strikes a surface does not need to be absorbed by that surface. Also, radiation that strikes a surface after being reflected by other surfaces is not considered in the evaluation of view factors.

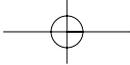
To develop a general expression for the view factor, consider two differential surfaces  $dA_1$  and  $dA_2$  on two arbitrarily oriented surfaces  $A_1$  and  $A_2$ , respectively, as shown in Fig. 22-2. The distance between  $dA_1$  and  $dA_2$  is  $r$ , and the angles between the normals of the surfaces and the line that connects  $dA_1$  and  $dA_2$  are  $\theta_1$  and  $\theta_2$ , respectively. Surface 1 emits and reflects radiation diffusely in all directions with a constant intensity of  $I_1$ , and the solid angle subtended by  $dA_2$  when viewed by  $dA_1$  is  $d\omega_{21}$ .

The rate at which radiation leaves  $dA_1$  in the direction of  $\theta_1$  is  $I_1 \cos \theta_1 dA_1$ . Noting that  $d\omega_{21} = dA_2 \cos \theta_2 / r^2$ , the portion of this radiation that strikes  $dA_2$  is

$$\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 \cos \theta_1 dA_1 d\omega_{21} = I_1 \cos \theta_1 dA_1 \frac{dA_2 \cos \theta_2}{r^2} \quad (22-1)$$

**FIGURE 22-2**

Geometry for the determination of the view factor between two surfaces.



The total rate at which radiation leaves  $dA_1$  (via emission and reflection) in all directions is the radiosity (which is  $J_1 = \pi I_1$ ) times the surface area,

$$\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1 \quad (22-2)$$

Then the *differential view factor*  $dF_{dA_1 \rightarrow dA_2}$ , which is the fraction of radiation leaving  $dA_1$  that strikes  $dA_2$  directly, becomes

$$dF_{dA_1 \rightarrow dA_2} = \frac{\dot{Q}_{dA_1 \rightarrow dA_2}}{\dot{Q}_{dA_1}} = \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_2 \quad (22-3)$$

The differential view factor  $dF_{dA_2 \rightarrow dA_1}$  can be determined from Eq. 22-3 by interchanging the subscripts 1 and 2.

The view factor from a differential area  $dA_1$  to a finite area  $A_2$  can be determined from the fact that the fraction of radiation leaving  $dA_1$  that strikes  $A_2$  is the sum of the fractions of radiation striking the differential areas  $dA_2$ . Therefore, the view factor  $F_{dA_1 \rightarrow A_2}$  is determined by integrating  $dF_{dA_1 \rightarrow dA_2}$  over  $A_2$ ,

$$F_{dA_1 \rightarrow A_2} = \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_2 \quad (22-4)$$

The total rate at which radiation leaves the entire  $A_1$  (via emission and reflection) in all directions is

$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1 \quad (22-5)$$

The portion of this radiation that strikes  $dA_2$  is determined by considering the radiation that leaves  $dA_1$  and strikes  $dA_2$  (given by Eq. 22-1), and integrating it over  $A_1$ ,

$$\dot{Q}_{A_1 \rightarrow dA_2} = \int_{A_1} \dot{Q}_{dA_1 \rightarrow dA_2} = \int_{A_1} \frac{I_1 \cos \theta_1 \cos \theta_2 dA_2}{r^2} dA_1 \quad (22-6)$$

Integration of this relation over  $A_2$  gives the radiation that strikes the entire  $A_2$ ,

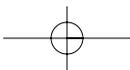
$$\dot{Q}_{A_1 \rightarrow A_2} = \int_{A_2} \dot{Q}_{A_1 \rightarrow dA_2} = \int_{A_2} \int_{A_1} \frac{I_1 \cos \theta_1 \cos \theta_2}{r^2} dA_1 dA_2 \quad (22-7)$$

Dividing this by the total radiation leaving  $A_1$  (from Eq. 22-5) gives the fraction of radiation leaving  $A_1$  that strikes  $A_2$ , which is the view factor  $F_{A_1 \rightarrow A_2}$  (or  $F_{12}$  for short),

$$F_{12} = F_{A_1 \rightarrow A_2} = \frac{\dot{Q}_{A_1 \rightarrow A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 dA_2 \quad (22-8)$$

The view factor  $F_{A_2 \rightarrow A_1}$  is readily determined from Eq. 22-8 by interchanging the subscripts 1 and 2,

$$F_{21} = F_{A_2 \rightarrow A_1} = \frac{\dot{Q}_{A_2 \rightarrow A_1}}{\dot{Q}_{A_2}} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 dA_2 \quad (22-9)$$



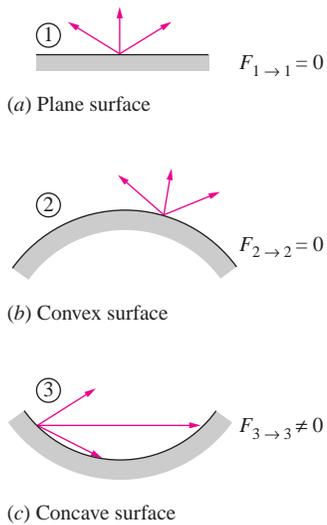


FIGURE 22-3

The view factor from a surface to itself is *zero* for *plane* or *convex* surfaces and *nonzero* for *concave* surfaces.

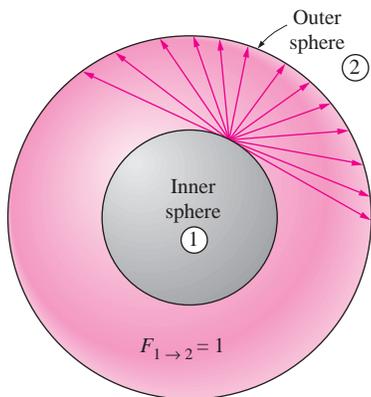


FIGURE 22-4

In a geometry that consists of two concentric spheres, the view factor  $F_{1 \rightarrow 2} = 1$  since the entire radiation leaving the surface of the smaller sphere will be intercepted by the larger sphere.

Note that  $I_1$  is constant but  $r$ ,  $\theta_1$ , and  $\theta_2$  are variables. Also, integrations can be performed in any order since the integration limits are constants. These relations confirm that the view factor between two surfaces depends on their relative orientation and the distance between them.

Combining Eqs. 22-8 and 22-9 after multiplying the former by  $A_1$  and the latter by  $A_2$  gives

$$A_1 F_{12} = A_2 F_{21} \quad (22-10)$$

which is known as the **reciprocity relation** for view factors. It allows the calculation of a view factor from a knowledge of the other.

The view factor relations developed above are applicable to any two surfaces  $i$  and  $j$  provided that the surfaces are diffuse emitters and diffuse reflectors (so that the assumption of constant intensity is valid). For the special case of  $j = i$ , we have

$$F_{i \rightarrow i} = \text{the fraction of radiation leaving surface } i \text{ that strikes itself directly}$$

Noting that in the absence of strong electromagnetic fields radiation beams travel in straight paths, the view factor from a surface to itself will be zero unless the surface “sees” itself. Therefore,  $F_{i \rightarrow i} = 0$  for *plane* or *convex* surfaces and  $F_{i \rightarrow i} \neq 0$  for *concave* surfaces, as illustrated in Fig. 22-3.

The value of the view factor ranges between *zero* and *one*. The limiting case  $F_{i \rightarrow j} = 0$  indicates that the two surfaces do not have a direct view of each other, and thus radiation leaving surface  $i$  cannot strike surface  $j$  directly. The other limiting case  $F_{i \rightarrow j} = 1$  indicates that surface  $j$  completely surrounds surface  $i$ , so that the entire radiation leaving surface  $i$  is intercepted by surface  $j$ . For example, in a geometry consisting of two concentric spheres, the entire radiation leaving the surface of the smaller sphere (surface 1) will strike the larger sphere (surface 2), and thus  $F_{1 \rightarrow 2} = 1$ , as illustrated in Fig. 22-4.

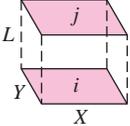
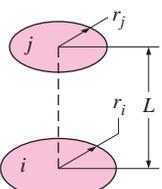
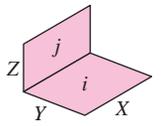
The view factor has proven to be very useful in radiation analysis because it allows us to express the *fraction of radiation* leaving a surface that strikes another surface in terms of the orientation of these two surfaces relative to each other. The underlying assumption in this process is that the radiation a surface receives from a source is directly proportional to the angle the surface subtends when viewed from the source. This would be the case only if the radiation coming off the source is *uniform* in all directions throughout its surface and the medium between the surfaces does not *absorb*, *emit*, or *scatter* radiation. That is, it will be the case when the surfaces are *isothermal* and *diffuse* emitters and reflectors and the surfaces are separated by a *non-participating* medium such as a vacuum or air.

The view factor  $F_{1 \rightarrow 2}$  between two surfaces  $A_1$  and  $A_2$  can be determined in a systematic manner first by expressing the view factor between two differential areas  $dA_1$  and  $dA_2$  in terms of the spatial variables and then by performing the necessary integrations. However, this approach is not practical, since, even for simple geometries, the resulting integrations are usually very complex and difficult to perform.

View factors for hundreds of common geometries are evaluated and the results are given in analytical, graphical, and tabular form in several publications. View factors for selected geometries are given in Tables 22-1 and 22-2 in *analytical* form and in Figs. 22-5 to 22-8 in *graphical* form. The view

TABLE 22-1

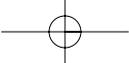
View factor expressions for some common geometries of finite size (3D)

Geometry	Relation
<p>Aligned parallel rectangles</p> 	$\bar{X} = X/L \text{ and } \bar{Y} = Y/L$ $F_{i \rightarrow j} = \frac{2}{\pi \bar{X} \bar{Y}} \left\{ \ln \left[ \frac{(1 + \bar{X}^2)(1 + \bar{Y}^2)}{1 + \bar{X}^2 + \bar{Y}^2} \right]^{1/2} \right.$ $+ \bar{X}(1 + \bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1 + \bar{Y}^2)^{1/2}}$ $+ \bar{Y}(1 + \bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1 + \bar{X}^2)^{1/2}}$ $\left. - \bar{X} \tan^{-1} \bar{X} - \bar{Y} \tan^{-1} \bar{Y} \right\}$
<p>Coaxial parallel disks</p> 	$R_i = r_i/L \text{ and } R_j = r_j/L$ $S = 1 + \frac{1 + R_j^2}{R_i^2}$ $F_{i \rightarrow j} = \frac{1}{2} \left\{ S - \left[ S^2 - 4 \left( \frac{r_j}{r_i} \right)^2 \right]^{1/2} \right\}$
<p>Perpendicular rectangles with a common edge</p> 	$H = Z/X \text{ and } W = Y/X$ $F_{i \rightarrow j} = \frac{1}{\pi W} \left( W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} \right.$ $\left. - (H^2 + W^2)^{1/2} \tan^{-1} \frac{1}{(H^2 + W^2)^{1/2}} \right.$ $+ \frac{1}{4} \ln \left\{ \frac{(1 + W^2)(1 + H^2)}{1 + W^2 + H^2} \right.$ $\times \left[ \frac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{W^2}$ $\left. \times \left[ \frac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \right]^{H^2} \right\}$

factors in Table 22-1 are for three-dimensional geometries. The view factors in Table 22-2, on the other hand, are for geometries that are *infinitely long* in the direction perpendicular to the plane of the paper and are therefore two-dimensional.

## 22-2 ■ VIEW FACTOR RELATIONS

Radiation analysis on an enclosure consisting of  $N$  surfaces requires the evaluation of  $N^2$  view factors, and this evaluation process is probably the most time-consuming part of a radiation analysis. However, it is neither practical nor necessary to evaluate all of the view factors directly. Once a sufficient number of view factors are available, the rest of them can be determined by utilizing some fundamental relations for view factors, as discussed next.



**TABLE 22-2**

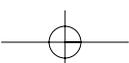
View factor expressions for some infinitely long (2D) geometries

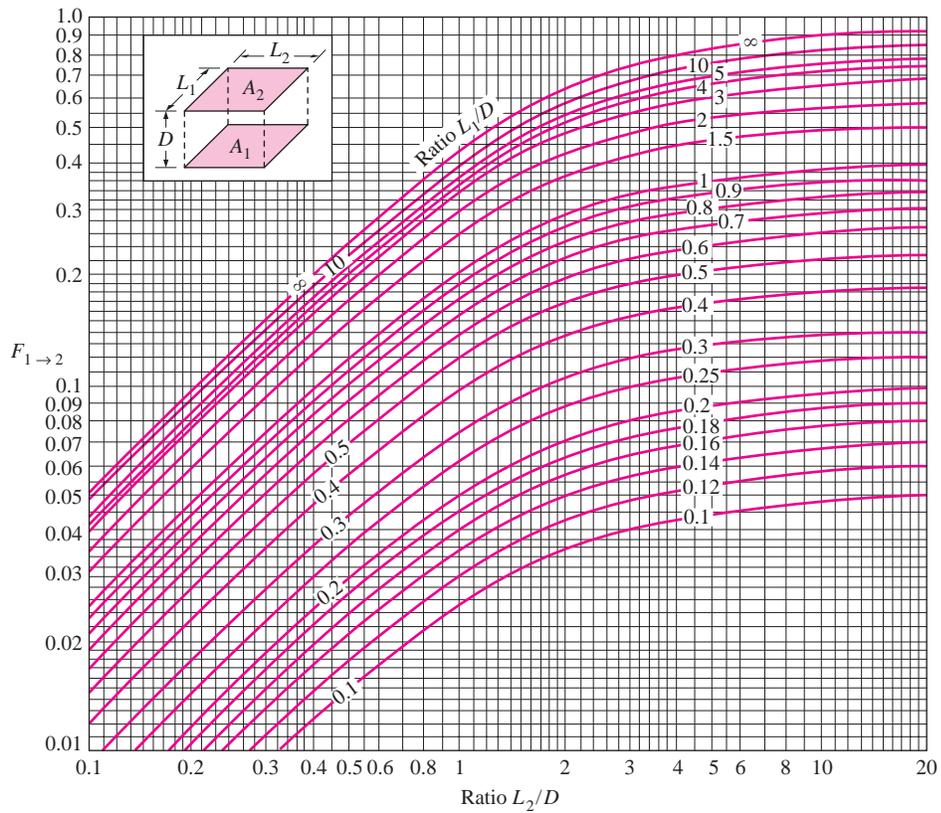
Geometry	Relation
Parallel plates with midlines connected by perpendicular line 	$W_i = w_i/L \text{ and } W_j = w_j/L$ $F_{i \rightarrow j} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_i}$
Inclined plates of equal width and with a common edge 	$F_{i \rightarrow j} = 1 - \sin \frac{1}{2} \alpha$
Perpendicular plates with a common edge 	$F_{i \rightarrow j} = \frac{1}{2} \left\{ 1 + \frac{w_j}{w_i} - \left[ 1 + \left( \frac{w_j}{w_i} \right)^2 \right]^{1/2} \right\}$
Three-sided enclosure 	$F_{i \rightarrow j} = \frac{w_i + w_j - w_k}{2w_i}$
Infinite plane and row of cylinders 	$F_{i \rightarrow j} = 1 - \left[ 1 - \left( \frac{D}{s} \right)^2 \right]^{1/2} + \frac{D}{s} \tan^{-1} \left( \frac{s^2 - D^2}{D^2} \right)^{1/2}$

## 1 The Reciprocity Relation

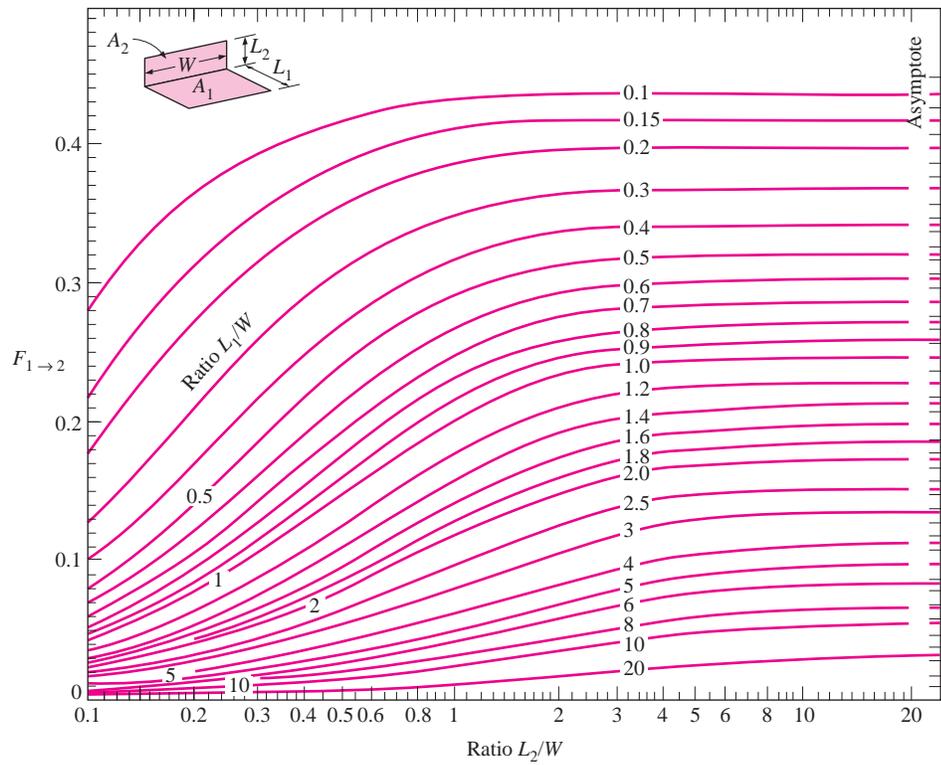
The view factors  $F_{i \rightarrow j}$  and  $F_{j \rightarrow i}$  are *not* equal to each other unless the areas of the two surfaces are. That is,

$$\begin{aligned}
 F_{j \rightarrow i} &= F_{i \rightarrow j} & \text{when} & & A_i &= A_j \\
 F_{j \rightarrow i} &\neq F_{i \rightarrow j} & \text{when} & & A_i &\neq A_j
 \end{aligned}$$

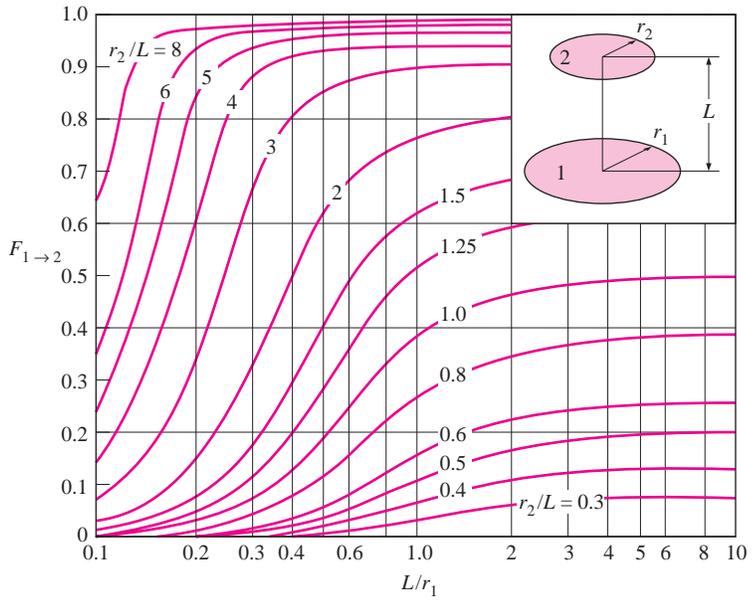
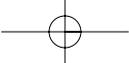




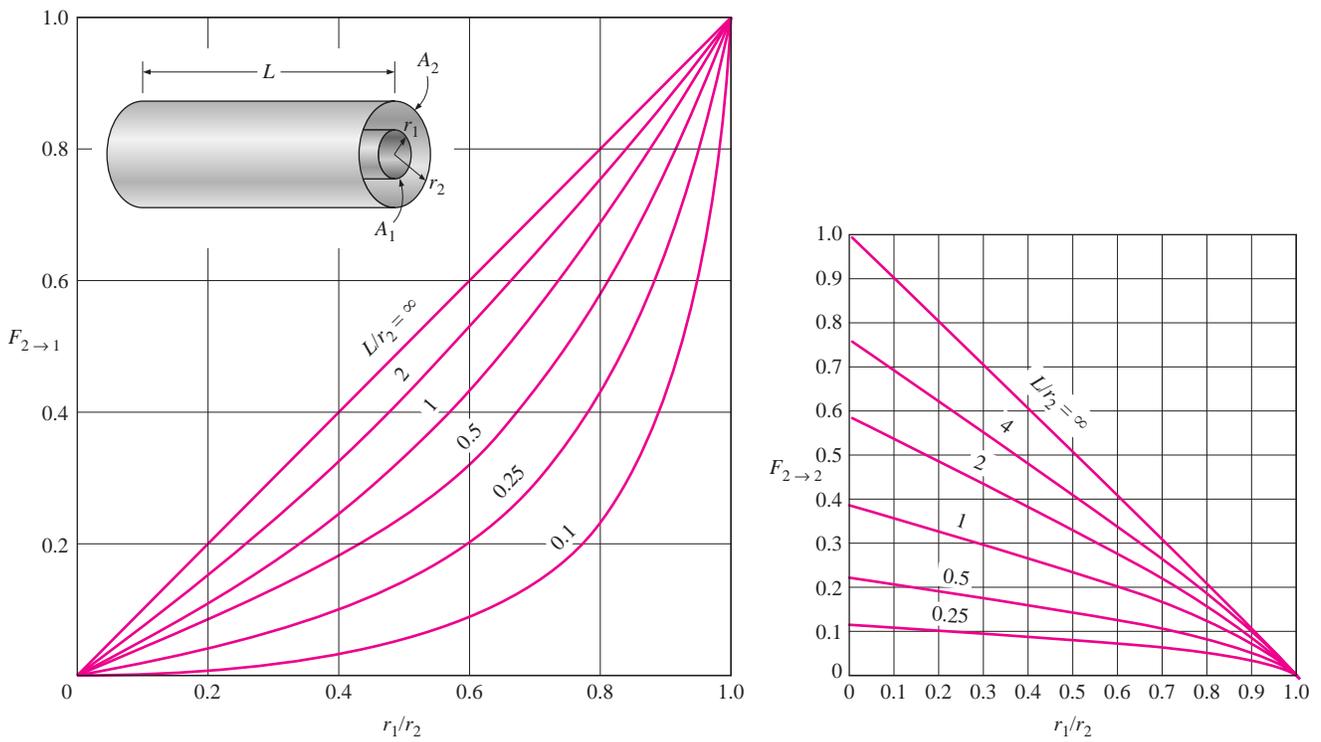
**FIGURE 22-5**  
View factor between two aligned parallel rectangles of equal size.



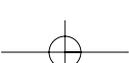
**FIGURE 22-6**  
View factor between two perpendicular rectangles with a common edge.

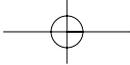


**FIGURE 22-7**  
View factor between two coaxial parallel disks.



**FIGURE 22-8**  
View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder; (b) outer cylinder to itself.





We have shown earlier the pair of view factors  $F_{i \rightarrow j}$  and  $F_{j \rightarrow i}$  are related to each other by

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} \quad (22-11)$$

This relation is referred to as the **reciprocity relation** or the **reciprocity rule**, and it enables us to determine the counterpart of a view factor from a knowledge of the view factor itself and the areas of the two surfaces. When determining the pair of view factors  $F_{i \rightarrow j}$  and  $F_{j \rightarrow i}$ , it makes sense to evaluate first the easier one directly and then the more difficult one by applying the reciprocity relation.

## 2 The Summation Rule

The radiation analysis of a surface normally requires the consideration of the radiation coming in or going out in all directions. Therefore, most radiation problems encountered in practice involve enclosed spaces. When formulating a radiation problem, we usually form an *enclosure* consisting of the surfaces interacting radiatively. Even openings are treated as imaginary surfaces with radiation properties equivalent to those of the opening.

The conservation of energy principle requires that the entire radiation leaving any surface  $i$  of an enclosure be intercepted by the surfaces of the enclosure. Therefore, *the sum of the view factors from surface  $i$  of an enclosure to all surfaces of the enclosure, including to itself, must equal unity*. This is known as the **summation rule** for an enclosure and is expressed as (Fig. 22-9)

$$\sum_{j=1}^N F_{i \rightarrow j} = 1 \quad (22-12)$$

where  $N$  is the number of surfaces of the enclosure. For example, applying the summation rule to surface 1 of a three-surface enclosure yields

$$\sum_{j=1}^3 F_{1 \rightarrow j} = F_{1 \rightarrow 1} + F_{1 \rightarrow 2} + F_{1 \rightarrow 3} = 1$$

The summation rule can be applied to each surface of an enclosure by varying  $i$  from 1 to  $N$ . Therefore, the summation rule applied to each of the  $N$  surfaces of an enclosure gives  $N$  relations for the determination of the view factors. Also, the reciprocity rule gives  $\frac{1}{2} N(N - 1)$  additional relations. Then the total number of view factors that need to be evaluated directly for an  $N$ -surface enclosure becomes

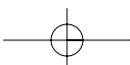
$$N^2 - [N + \frac{1}{2} N(N - 1)] = \frac{1}{2} N(N - 1)$$

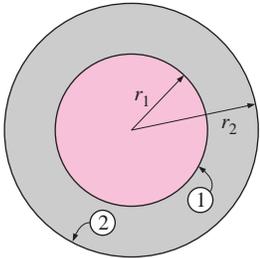
For example, for a six-surface enclosure, we need to determine only  $\frac{1}{2} \times 6(6 - 1) = 15$  of the  $6^2 = 36$  view factors directly. The remaining 21 view factors can be determined from the 21 equations that are obtained by applying the reciprocity and the summation rules.



**FIGURE 22-9**

Radiation leaving any surface  $i$  of an enclosure must be intercepted completely by the surfaces of the enclosure. Therefore, the sum of the view factors from surface  $i$  to each one of the surfaces of the enclosure must be unity.



**FIGURE 22-10**

The geometry considered in Example 22-1.

### EXAMPLE 22-1 View Factors Associated with Two Concentric Spheres

Determine the view factors associated with an enclosure formed by two spheres, shown in Fig. 22-10.

**SOLUTION** The view factors associated with two concentric spheres are to be determined.

**Assumptions** The surfaces are diffuse emitters and reflectors.

**Analysis** The outer surface of the smaller sphere (surface 1) and inner surface of the larger sphere (surface 2) form a two-surface enclosure. Therefore,  $N = 2$  and this enclosure involves  $N^2 = 2^2 = 4$  view factors, which are  $F_{11}$ ,  $F_{12}$ ,  $F_{21}$ , and  $F_{22}$ . In this two-surface enclosure, we need to determine only

$$\frac{1}{2}N(N-1) = \frac{1}{2} \times 2(2-1) = 1$$

view factor directly. The remaining three view factors can be determined by the application of the summation and reciprocity rules. But it turns out that we can determine not only one but *two* view factors directly in this case by a simple *inspection*:

$$\begin{aligned} F_{11} &= 0, & \text{since no radiation leaving surface 1 strikes itself} \\ F_{12} &= 1, & \text{since all radiation leaving surface 1 strikes surface 2} \end{aligned}$$

Actually it would be sufficient to determine only one of these view factors by inspection, since we could always determine the other one from the summation rule applied to surface 1 as  $F_{11} + F_{12} = 1$ .

The view factor  $F_{21}$  is determined by applying the reciprocity relation to surfaces 1 and 2:

$$A_1 F_{12} = A_2 F_{21}$$

which yields

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

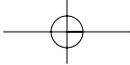
Finally, the view factor  $F_{22}$  is determined by applying the summation rule to surface 2:

$$F_{21} + F_{22} = 1$$

and thus

$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

**Discussion** Note that when the outer sphere is much larger than the inner sphere ( $r_2 \gg r_1$ ),  $F_{22}$  approaches one. This is expected, since the fraction of radiation leaving the outer sphere that is intercepted by the inner sphere will be negligible in that case. Also note that the two spheres considered above do not need to be concentric. However, the radiation analysis will be most accurate for the case of concentric spheres, since the radiation is most likely to be uniform on the surfaces in that case.



### 3 The Superposition Rule

Sometimes the view factor associated with a given geometry is not available in standard tables and charts. In such cases, it is desirable to express the given geometry as the sum or difference of some geometries with known view factors, and then to apply the **superposition rule**, which can be expressed as *the view factor from a surface  $i$  to a surface  $j$  is equal to the sum of the view factors from surface  $i$  to the parts of surface  $j$* . Note that the reverse of this is not true. That is, the view factor from a surface  $j$  to a surface  $i$  is *not* equal to the sum of the view factors from the parts of surface  $j$  to surface  $i$ .

Consider the geometry in Fig. 22–11, which is infinitely long in the direction perpendicular to the plane of the paper. The radiation that leaves surface 1 and strikes the combined surfaces 2 and 3 is equal to the sum of the radiation that strikes surfaces 2 and 3. Therefore, the view factor from surface 1 to the combined surfaces of 2 and 3 is

$$F_{1 \rightarrow (2,3)} = F_{1 \rightarrow 2} + F_{1 \rightarrow 3} \quad (22-13)$$

Suppose we need to find the view factor  $F_{1 \rightarrow 3}$ . A quick check of the view factor expressions and charts in this section will reveal that such a view factor cannot be evaluated directly. However, the view factor  $F_{1 \rightarrow 3}$  can be determined from Eq. 22–13 after determining both  $F_{1 \rightarrow 2}$  and  $F_{1 \rightarrow (2,3)}$  from the chart in Table 22–2. Therefore, it may be possible to determine some difficult view factors with relative ease by expressing one or both of the areas as the sum or differences of areas and then applying the superposition rule.

To obtain a relation for the view factor  $F_{(2,3) \rightarrow 1}$ , we multiply Eq. 22–13 by  $A_1$ ,

$$A_1 F_{1 \rightarrow (2,3)} = A_1 F_{1 \rightarrow 2} + A_1 F_{1 \rightarrow 3}$$

and apply the reciprocity relation to each term to get

$$(A_2 + A_3)F_{(2,3) \rightarrow 1} = A_2 F_{2 \rightarrow 1} + A_3 F_{3 \rightarrow 1}$$

or

$$F_{(2,3) \rightarrow 1} = \frac{A_2 F_{2 \rightarrow 1} + A_3 F_{3 \rightarrow 1}}{A_2 + A_3} \quad (22-14)$$

Areas that are expressed as the sum of more than two parts can be handled in a similar manner.

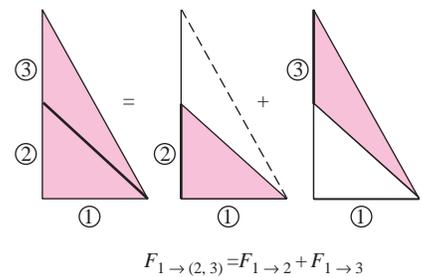


FIGURE 22–11

The view factor from a surface to a composite surface is equal to the sum of the view factors from the surface to the parts of the composite surface.

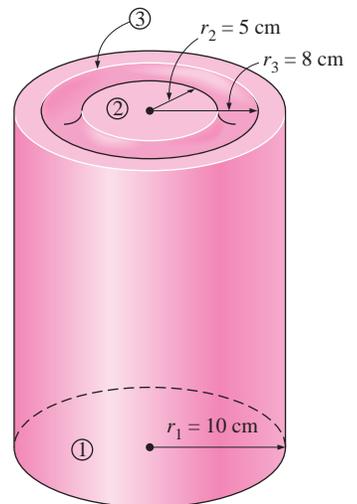
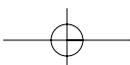


FIGURE 22–12

The cylindrical enclosure considered in Example 22–2.

#### EXAMPLE 22–2 Fraction of Radiation Leaving through an Opening

Determine the fraction of the radiation leaving the base of the cylindrical enclosure shown in Fig. 22–12 that escapes through a coaxial ring opening at its top surface. The radius and the length of the enclosure are  $r_1 = 10$  cm and  $L = 10$  cm, while the inner and outer radii of the ring are  $r_2 = 5$  cm and  $r_3 = 8$  cm, respectively.



**SOLUTION** The fraction of radiation leaving the base of a cylindrical enclosure through a coaxial ring opening at its top surface is to be determined.

**Assumptions** The base surface is a diffuse emitter and reflector.

**Analysis** We are asked to determine the fraction of the radiation leaving the base of the enclosure that escapes through an opening at the top surface. Actually, what we are asked to determine is simply the *view factor*  $F_{1 \rightarrow \text{ring}}$  from the base of the enclosure to the ring-shaped surface at the top.

We do not have an analytical expression or chart for view factors between a circular area and a coaxial ring, and so we cannot determine  $F_{1 \rightarrow \text{ring}}$  directly. However, we do have a chart for view factors between two coaxial parallel disks, and we can always express a ring in terms of disks.

Let the base surface of radius  $r_1 = 10$  cm be surface 1, the circular area of  $r_2 = 5$  cm at the top be surface 2, and the circular area of  $r_3 = 8$  cm be surface 3. Using the superposition rule, the view factor from surface 1 to surface 3 can be expressed as

$$F_{1 \rightarrow 3} = F_{1 \rightarrow 2} + F_{1 \rightarrow \text{ring}}$$

since surface 3 is the sum of surface 2 and the ring area. The view factors  $F_{1 \rightarrow 2}$  and  $F_{1 \rightarrow 3}$  are determined from the chart in Fig. 22-7.

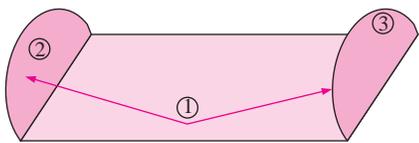
$$\frac{L}{r_1} = \frac{10 \text{ cm}}{10 \text{ cm}} = 1 \quad \text{and} \quad \frac{r_2}{L} = \frac{5 \text{ cm}}{10 \text{ cm}} = 0.5 \quad \xrightarrow{\text{(Fig. 22-7)}} \quad F_{1 \rightarrow 2} = 0.11$$

$$\frac{L}{r_1} = \frac{10 \text{ cm}}{10 \text{ cm}} = 1 \quad \text{and} \quad \frac{r_3}{L} = \frac{8 \text{ cm}}{10 \text{ cm}} = 0.8 \quad \xrightarrow{\text{(Fig. 22-7)}} \quad F_{1 \rightarrow 3} = 0.28$$

Therefore,

$$F_{1 \rightarrow \text{ring}} = F_{1 \rightarrow 3} - F_{1 \rightarrow 2} = 0.28 - 0.11 = \mathbf{0.17}$$

which is the desired result. Note that  $F_{1 \rightarrow 2}$  and  $F_{1 \rightarrow 3}$  represent the fractions of radiation leaving the base that strike the circular surfaces 2 and 3, respectively, and their difference gives the fraction that strikes the ring area.



$$F_{1 \rightarrow 2} = F_{1 \rightarrow 3}$$

(Also,  $F_{2 \rightarrow 1} = F_{3 \rightarrow 1}$ )

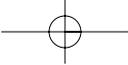
**FIGURE 22-13**

Two surfaces that are symmetric about a third surface will have the same view factor from the third surface.

## 4 The Symmetry Rule

The determination of the view factors in a problem can be simplified further if the geometry involved possesses some sort of symmetry. Therefore, it is good practice to check for the presence of any *symmetry* in a problem before attempting to determine the view factors directly. The presence of symmetry can be determined *by inspection*, keeping the definition of the view factor in mind. Identical surfaces that are oriented in an identical manner with respect to another surface will intercept identical amounts of radiation leaving that surface. Therefore, the **symmetry rule** can be expressed as *two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface* (Fig. 22-13).

The symmetry rule can also be expressed as *if the surfaces  $j$  and  $k$  are symmetric about the surface  $i$  then  $F_{i \rightarrow j} = F_{i \rightarrow k}$* . Using the reciprocity rule, we can show that the relation  $F_{j \rightarrow i} = F_{k \rightarrow i}$  is also true in this case.

**EXAMPLE 22-3 View Factors Associated with a Tetragon**

Determine the view factors from the base of the pyramid shown in Fig. 22-14 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

**SOLUTION** The view factors from the base of a pyramid to each of its four side surfaces for the case of a square base are to be determined.

**Assumptions** The surfaces are diffuse emitters and reflectors.

**Analysis** The base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3, 4, and 5) form a five-surface enclosure. The first thing we notice about this enclosure is its symmetry. The four side surfaces are symmetric about the base surface. Then, from the *symmetry rule*, we have

$$F_{12} = F_{13} = F_{14} = F_{15}$$

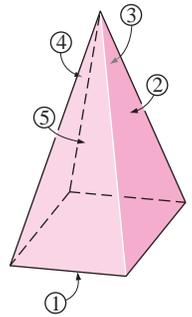
Also, the *summation rule* applied to surface 1 yields

$$\sum_{j=1}^5 F_{1j} = F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

However,  $F_{11} = 0$ , since the base is a *flat* surface. Then the two relations above yield

$$F_{12} = F_{13} = F_{14} = F_{15} = \mathbf{0.25}$$

**Discussion** Note that each of the four side surfaces of the pyramid receive one-fourth of the entire radiation leaving the base surface, as expected. Also note that the presence of symmetry greatly simplified the determination of the view factors.



**FIGURE 22-14**  
The pyramid considered in Example 22-3.

**EXAMPLE 22-4 View Factors Associated with a Triangular Duct**

Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross section is given in Fig. 22-15.

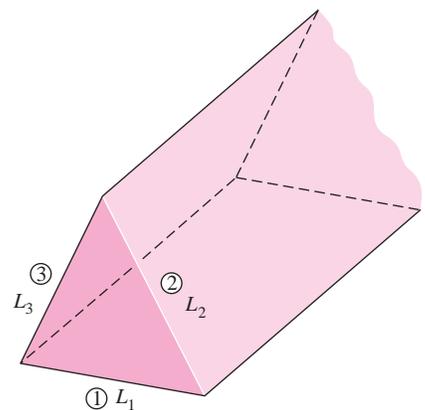
**SOLUTION** The view factors associated with an infinitely long triangular duct are to be determined.

**Assumptions** The surfaces are diffuse emitters and reflectors.

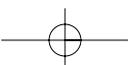
**Analysis** The widths of the sides of the triangular cross section of the duct are  $L_1$ ,  $L_2$ , and  $L_3$ , and the surface areas corresponding to them are  $A_1$ ,  $A_2$ , and  $A_3$ , respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three-surface enclosure,  $N = 3$ .

This enclosure involves  $N^2 = 3^2 = 9$  view factors, and we need to determine

$$\frac{1}{2}N(N-1) = \frac{1}{2} \times 3(3-1) = 3$$



**FIGURE 22-15**  
The infinitely long triangular duct considered in Example 22-4.



of these view factors directly. Fortunately, we can determine all three of them by inspection to be

$$F_{11} = F_{22} = F_{33} = 0$$

since all three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules.

Applying the summation rule to each of the three surfaces gives

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{21} + F_{22} + F_{23} = 1$$

$$F_{31} + F_{32} + F_{33} = 1$$

Noting that  $F_{11} = F_{22} = F_{33} = 0$  and multiplying the first equation by  $A_1$ , the second by  $A_2$ , and the third by  $A_3$  gives

$$A_1 F_{12} + A_1 F_{13} = A_1$$

$$A_2 F_{21} + A_2 F_{23} = A_2$$

$$A_3 F_{31} + A_3 F_{32} = A_3$$

Finally, applying the three reciprocity relations  $A_1 F_{12} = A_2 F_{21}$ ,  $A_1 F_{13} = A_3 F_{31}$ , and  $A_2 F_{23} = A_3 F_{32}$  gives

$$A_1 F_{12} + A_1 F_{13} = A_1$$

$$A_1 F_{12} + A_2 F_{23} = A_2$$

$$A_1 F_{13} + A_2 F_{23} = A_3$$

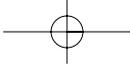
This is a set of three algebraic equations with three unknowns, which can be solved to obtain

$$\begin{aligned} F_{12} &= \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1} \\ F_{13} &= \frac{A_1 + A_3 - A_2}{2A_1} = \frac{L_1 + L_3 - L_2}{2L_1} \\ F_{23} &= \frac{A_2 + A_3 - A_1}{2A_2} = \frac{L_2 + L_3 - L_1}{2L_2} \end{aligned} \quad (22-15)$$

**Discussion** Note that we have replaced the areas of the side surfaces by their corresponding widths for simplicity, since  $A = Ls$  and the length  $s$  can be factored out and canceled. We can generalize this result as *the view factor from a surface of a very long triangular duct to another surface is equal to the sum of the widths of these two surfaces minus the width of the third surface, divided by twice the width of the first surface.*

## View Factors between Infinitely Long Surfaces: The Crossed-Strings Method

Many problems encountered in practice involve geometries of constant cross section such as channels and ducts that are *very long* in one direction relative



to the other directions. Such geometries can conveniently be considered to be *two-dimensional*, since any radiation interaction through their end surfaces will be negligible. These geometries can subsequently be modeled as being *infinitely long*, and the view factor between their surfaces can be determined by the amazingly simple *crossed-strings method* developed by H. C. Hottel in the 1950s. The surfaces of the geometry do not need to be flat; they can be convex, concave, or any irregular shape.

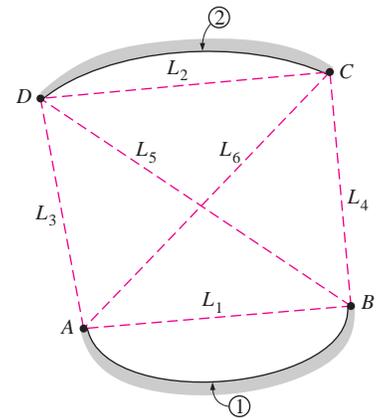
To demonstrate this method, consider the geometry shown in Fig. 22–16, and let us try to find the view factor  $F_{1 \rightarrow 2}$  between surfaces 1 and 2. The first thing we do is identify the endpoints of the surfaces (the points A, B, C, and D) and connect them to each other with tightly stretched strings, which are indicated by dashed lines. Hottel has shown that the view factor  $F_{1 \rightarrow 2}$  can be expressed in terms of the lengths of these stretched strings, which are straight lines, as

$$F_{1 \rightarrow 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \quad (22-16)$$

Note that  $L_5 + L_6$  is the sum of the lengths of the *crossed strings*, and  $L_3 + L_4$  is the sum of the lengths of the *uncrossed strings* attached to the endpoints. Therefore, Hottel’s crossed-strings method can be expressed verbally as

$$F_{i \rightarrow j} = \frac{\Sigma (\text{Crossed strings}) - \Sigma (\text{Uncrossed strings})}{2 \times (\text{String on surface } i)} \quad (22-17)$$

The crossed-strings method is applicable even when the two surfaces considered share a common edge, as in a triangle. In such cases, the common edge can be treated as an imaginary string of zero length. The method can also be applied to surfaces that are partially blocked by other surfaces by allowing the strings to bend around the blocking surfaces.



**FIGURE 22-16**  
Determination of the view factor  $F_{1 \rightarrow 2}$  by the application of the crossed-strings method.

**EXAMPLE 22-5 The Crossed-Strings Method for View Factors**

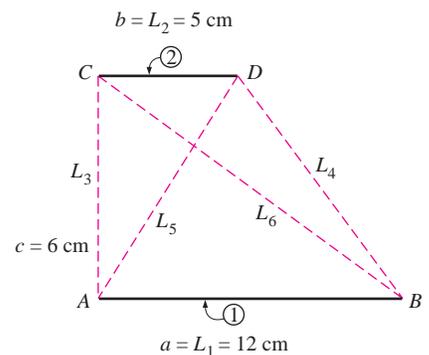
Two infinitely long parallel plates of widths  $a = 12$  cm and  $b = 5$  cm are located a distance  $c = 6$  cm apart, as shown in Fig. 22–17. (a) Determine the view factor  $F_{1 \rightarrow 2}$  from surface 1 to surface 2 by using the crossed-strings method. (b) Derive the crossed-strings formula by forming triangles on the given geometry and using Eq. 22–15 for view factors between the sides of triangles.

**SOLUTION** The view factors between two infinitely long parallel plates are to be determined using the crossed-strings method, and the formula for the view factor is to be derived.

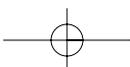
**Assumptions** The surfaces are diffuse emitters and reflectors.

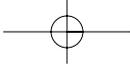
**Analysis** (a) First we label the endpoints of both surfaces and draw straight dashed lines between the endpoints, as shown in Fig. 22–17. Then we identify the crossed and uncrossed strings and apply the crossed-strings method (Eq. 22–17) to determine the view factor  $F_{1 \rightarrow 2}$ :

$$F_{1 \rightarrow 2} = \frac{\Sigma (\text{Crossed strings}) - \Sigma (\text{Uncrossed strings})}{2 \times (\text{String on surface 1})} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$



**FIGURE 22-17**  
The two infinitely long parallel plates considered in Example 22–5.





where

$$\begin{aligned} L_1 &= a = 12 \text{ cm} & L_4 &= \sqrt{7^2 + 6^2} = 9.22 \text{ cm} \\ L_2 &= b = 5 \text{ cm} & L_5 &= \sqrt{5^2 + 6^2} = 7.81 \text{ cm} \\ L_3 &= c = 6 \text{ cm} & L_6 &= \sqrt{12^2 + 6^2} = 13.42 \text{ cm} \end{aligned}$$

Substituting,

$$F_{1 \rightarrow 2} = \frac{[(7.81 + 13.42) - (6 + 9.22)] \text{ cm}}{2 \times 12 \text{ cm}} = \mathbf{0.250}$$

(b) The geometry is infinitely long in the direction perpendicular to the plane of the paper, and thus the two plates (surfaces 1 and 2) and the two openings (imaginary surfaces 3 and 4) form a four-surface enclosure. Then applying the summation rule to surface 1 yields

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

But  $F_{11} = 0$  since it is a flat surface. Therefore,

$$F_{12} = 1 - F_{13} - F_{14}$$

where the view factors  $F_{13}$  and  $F_{14}$  can be determined by considering the triangles  $ABC$  and  $ABD$ , respectively, and applying Eq. 22-15 for view factors between the sides of triangles. We obtain

$$F_{13} = \frac{L_1 + L_3 - L_6}{2L_1}, \quad F_{14} = \frac{L_1 + L_4 - L_5}{2L_1}$$

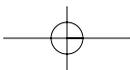
Substituting,

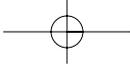
$$\begin{aligned} F_{12} &= 1 - \frac{L_1 + L_3 - L_6}{2L_1} - \frac{L_1 + L_4 - L_5}{2L_1} \\ &= \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \end{aligned}$$

which is the desired result. This is also a miniproof of the crossed-strings method for the case of two infinitely long plain parallel surfaces.

## 22-3 ■ RADIATION HEAT TRANSFER: BLACK SURFACES

So far, we have considered the nature of radiation, the radiation properties of materials, and the view factors, and we are now in a position to consider the rate of heat transfer between surfaces by radiation. The analysis of radiation exchange between surfaces, in general, is complicated because of reflection: a radiation beam leaving a surface may be reflected several times, with partial reflection occurring at each surface, before it is completely absorbed. The analysis is simplified greatly when the surfaces involved can be approximated





as blackbodies because of the absence of reflection. In this section, we consider radiation exchange between *black surfaces* only; we will extend the analysis to reflecting surfaces in the next section.

Consider two black surfaces of arbitrary shape maintained at uniform temperatures  $T_1$  and  $T_2$ , as shown in Fig. 22–18. Recognizing that radiation leaves a black surface at a rate of  $E_b = \sigma T^4$  per unit surface area and that the view factor  $F_{1 \rightarrow 2}$  represents the fraction of radiation leaving surface 1 that strikes surface 2, the *net* rate of radiation heat transfer from surface 1 to surface 2 can be expressed as

$$\begin{aligned} \dot{Q}_{1 \rightarrow 2} &= \left( \begin{array}{l} \text{Radiation leaving} \\ \text{the entire surface 1} \\ \text{that strikes surface 2} \end{array} \right) - \left( \begin{array}{l} \text{Radiation leaving} \\ \text{the entire surface 2} \\ \text{that strikes surface 1} \end{array} \right) \\ &= A_1 E_{b1} F_{1 \rightarrow 2} - A_2 E_{b2} F_{2 \rightarrow 1} \quad (\text{W}) \end{aligned} \quad (22-18)$$

Applying the reciprocity relation  $A_1 F_{1 \rightarrow 2} = A_2 F_{2 \rightarrow 1}$  yields

$$\dot{Q}_{1 \rightarrow 2} = A_1 F_{1 \rightarrow 2} \sigma (T_1^4 - T_2^4) \quad (\text{W}) \quad (22-19)$$

which is the desired relation. A negative value for  $\dot{Q}_{1 \rightarrow 2}$  indicates that net radiation heat transfer is from surface 2 to surface 1.

Now consider an *enclosure* consisting of  $N$  *black* surfaces maintained at specified temperatures. The *net* radiation heat transfer *from* any surface  $i$  of this enclosure is determined by adding up the net radiation heat transfers from surface  $i$  to each of the surfaces of the enclosure:

$$\dot{Q}_i = \sum_{j=1}^N \dot{Q}_{i \rightarrow j} = \sum_{j=1}^N A_i F_{i \rightarrow j} \sigma (T_i^4 - T_j^4) \quad (\text{W}) \quad (22-20)$$

Again a negative value for  $\dot{Q}_i$  indicates that net radiation heat transfer is *to* surface  $i$  (i.e., surface  $i$  *gains* radiation energy instead of losing). Also, the net heat transfer from a surface to itself is zero, regardless of the shape of the surface.

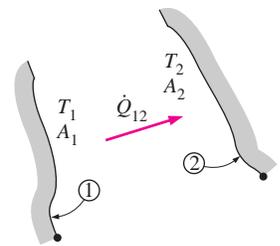


FIGURE 22–18

Two general black surfaces maintained at uniform temperatures  $T_1$  and  $T_2$ .

### EXAMPLE 22–6 Radiation Heat Transfer in a Black Furnace

Consider the 5-m  $\times$  5-m  $\times$  5-m cubical furnace shown in Fig. 22–19, whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at uniform temperatures of 800 K, 1500 K, and 500 K, respectively. Determine (a) the net rate of radiation heat transfer between the base and the side surfaces, (b) the net rate of radiation heat transfer between the base and the top surface, and (c) the net radiation heat transfer from the base surface.

**SOLUTION** The surfaces of a cubical furnace are black and are maintained at uniform temperatures. The net rate of radiation heat transfer between the base and side surfaces, between the base and the top surface, and from the base surface are to be determined.

**Assumptions** The surfaces are black and isothermal.

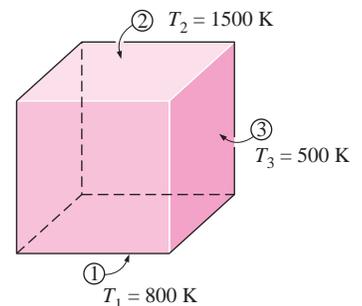
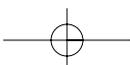


FIGURE 22–19

The cubical furnace of black surfaces considered in Example 22–6.



**Analysis** (a) Considering that the geometry involves six surfaces, we may be tempted at first to treat the furnace as a six-surface enclosure. However, the four side surfaces possess the same properties, and thus we can treat them as a single side surface in radiation analysis. We consider the base surface to be surface 1, the top surface to be surface 2, and the side surfaces to be surface 3. Then the problem reduces to determining  $\dot{Q}_{1 \rightarrow 3}$ ,  $\dot{Q}_{1 \rightarrow 2}$ , and  $\dot{Q}_1$ .

The net rate of radiation heat transfer  $\dot{Q}_{1 \rightarrow 3}$  from surface 1 to surface 3 can be determined from Eq. 22-19, since both surfaces involved are black, by replacing the subscript 2 by 3:

$$\dot{Q}_{1 \rightarrow 3} = A_1 F_{1 \rightarrow 3} \sigma (T_1^4 - T_3^4)$$

But first we need to evaluate the view factor  $F_{1 \rightarrow 3}$ . After checking the view factor charts and tables, we realize that we cannot determine this view factor directly. However, we can determine the view factor  $F_{1 \rightarrow 2}$  directly from Fig. 22-5 to be  $F_{1 \rightarrow 2} = 0.2$ , and we know that  $F_{1 \rightarrow 1} = 0$  since surface 1 is a plane. Then applying the summation rule to surface 1 yields

$$F_{1 \rightarrow 1} + F_{1 \rightarrow 2} + F_{1 \rightarrow 3} = 1$$

or

$$F_{1 \rightarrow 3} = 1 - F_{1 \rightarrow 1} - F_{1 \rightarrow 2} = 1 - 0 - 0.2 = 0.8$$

Substituting,

$$\begin{aligned} \dot{Q}_{1 \rightarrow 3} &= (25 \text{ m}^2)(0.8)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(800 \text{ K})^4 - (500 \text{ K})^4] \\ &= \mathbf{394 \times 10^3 \text{ W} = 394 \text{ kW}} \end{aligned}$$

(b) The net rate of radiation heat transfer  $\dot{Q}_{1 \rightarrow 2}$  from surface 1 to surface 2 is determined in a similar manner from Eq. 22-19 to be

$$\begin{aligned} \dot{Q}_{1 \rightarrow 2} &= A_1 F_{1 \rightarrow 2} \sigma (T_1^4 - T_2^4) \\ &= (25 \text{ m}^2)(0.2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(800 \text{ K})^4 - (1500 \text{ K})^4] \\ &= \mathbf{-1319 \times 10^3 \text{ W} = -1319 \text{ kW}} \end{aligned}$$

The negative sign indicates that net radiation heat transfer is from surface 2 to surface 1.

(c) The net radiation heat transfer from the base surface  $\dot{Q}_1$  is determined from Eq. 22-20 by replacing the subscript  $i$  by 1 and taking  $N = 3$ :

$$\begin{aligned} \dot{Q}_1 &= \sum_{j=1}^3 \dot{Q}_{1 \rightarrow j} = \dot{Q}_{1 \rightarrow 1} + \dot{Q}_{1 \rightarrow 2} + \dot{Q}_{1 \rightarrow 3} \\ &= 0 + (-1319 \text{ kW}) + (394 \text{ kW}) \\ &= \mathbf{-925 \text{ kW}} \end{aligned}$$

Again the negative sign indicates that net radiation heat transfer is *to* surface 1. That is, the base of the furnace is gaining net radiation at a rate of about 925 kW.

## 22-4 ■ RADIATION HEAT TRANSFER: DIFFUSE, GRAY SURFACES

The analysis of radiation transfer in enclosures consisting of black surfaces is relatively easy, as we have seen, but most enclosures encountered in practice involve nonblack surfaces, which allow multiple reflections to occur. Radiation analysis of such enclosures becomes very complicated unless some simplifying assumptions are made.

To make a simple radiation analysis possible, it is common to assume the surfaces of an enclosure to be *opaque*, *diffuse*, and *gray*. That is, the surfaces are nontransparent, they are diffuse emitters and diffuse reflectors, and their radiation properties are independent of wavelength. Also, each surface of the enclosure is *isothermal*, and both the incoming and outgoing radiation are *uniform* over each surface. But first we review the concept of radiosity discussed in Chap. 21.

### Radiosity

Surfaces emit radiation as well as reflect it, and thus the radiation leaving a surface consists of emitted and reflected parts. The calculation of radiation heat transfer between surfaces involves the *total* radiation energy streaming away from a surface, with no regard for its origin. The *total radiation energy leaving a surface per unit time and per unit area* is the **radiosity** and is denoted by  $J$  (Fig. 22-20).

For a surface  $i$  that is *gray* and *opaque* ( $\varepsilon_i = \alpha_i$  and  $\alpha_i + \rho_i = 1$ ), the radiosity can be expressed as

$$\begin{aligned} J_i &= \left( \begin{array}{c} \text{Radiation emitted} \\ \text{by surface } i \end{array} \right) + \left( \begin{array}{c} \text{Radiation reflected} \\ \text{by surface } i \end{array} \right) \\ &= \varepsilon_i E_{bi} + \rho_i G_i \\ &= \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i \quad (\text{W/m}^2) \end{aligned} \quad (22-21)$$

where  $E_{bi} = \sigma T_i^4$  is the blackbody emissive power of surface  $i$  and  $G_i$  is irradiation (i.e., the radiation energy incident on surface  $i$  per unit time per unit area).

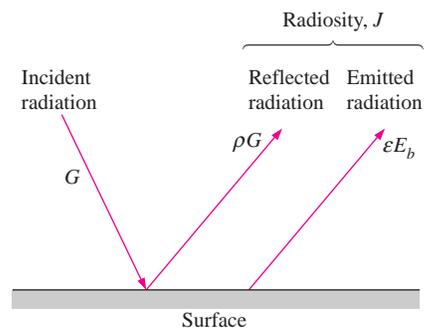
For a surface that can be approximated as a *blackbody* ( $\varepsilon_i = 1$ ), the radiosity relation reduces to

$$J_i = E_{bi} = \sigma T_i^4 \quad (\text{blackbody}) \quad (22-22)$$

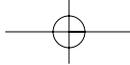
That is, *the radiosity of a blackbody is equal to its emissive power*. This is expected, since a blackbody does not reflect any radiation, and thus radiation coming from a blackbody is due to emission only.

### Net Radiation Heat Transfer to or from a Surface

During a radiation interaction, a surface *loses* energy by emitting radiation and *gains* energy by absorbing radiation emitted by other surfaces. A surface experiences a net gain or a net loss of energy, depending on which quantity is larger. The *net* rate of radiation heat transfer from a surface  $i$  of surface area  $A_i$  is denoted by  $\dot{Q}_i$  and is expressed as



**FIGURE 22-20**  
Radiosity represents the sum of the radiation energy emitted and reflected by a surface.



$$\begin{aligned} \dot{Q}_i &= \left( \begin{array}{l} \text{Radiation leaving} \\ \text{entire surface } i \end{array} \right) - \left( \begin{array}{l} \text{Radiation incident} \\ \text{on entire surface } i \end{array} \right) \\ &= A_i(J_i - G_i) \quad (\text{W}) \end{aligned} \quad (22-23)$$

Solving for  $G_i$  from Eq. 22-21 and substituting into Eq. 22-23 yields

$$\dot{Q}_i = A_i \left( J_i - \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i} \right) = \frac{A_i \epsilon_i}{1 - \epsilon_i} (E_{bi} - J_i) \quad (\text{W}) \quad (22-24)$$

In an electrical analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \quad (\text{W}) \quad (22-25)$$

where

$$R_i = \frac{1 - \epsilon_i}{A_i \epsilon_i} \quad (22-26)$$

is the **surface resistance** to radiation. The quantity  $E_{bi} - J_i$  corresponds to a *potential difference* and the net rate of radiation heat transfer corresponds to *current* in the electrical analogy, as illustrated in Fig. 22-21.

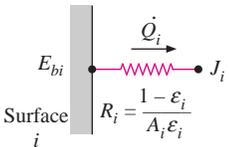
The direction of the net radiation heat transfer depends on the relative magnitudes of  $J_i$  (the radiosity) and  $E_{bi}$  (the emissive power of a blackbody at the temperature of the surface). It will be *from* the surface if  $E_{bi} > J_i$  and *to* the surface if  $J_i > E_{bi}$ . A negative value for  $\dot{Q}_i$  indicates that heat transfer is *to* the surface. All of this radiation energy gained must be removed from the other side of the surface through some mechanism if the surface temperature is to remain constant.

The surface resistance to radiation for a *blackbody* is *zero* since  $\epsilon_i = 1$  and  $J_i = E_{bi}$ . The net rate of radiation heat transfer in this case is determined directly from Eq. 22-23.

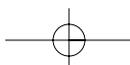
Some surfaces encountered in numerous practical heat transfer applications are modeled as being *adiabatic* since their back sides are well insulated and the net heat transfer through them is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiate energy as it gains, and thus  $\dot{Q}_i = 0$ . In such cases, the surface is said to *reradiate* all the radiation energy it receives, and such a surface is called a **reradiating surface**. Setting  $\dot{Q}_i = 0$  in Eq. 22-25 yields

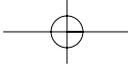
$$J_i = E_{bi} = \sigma T_i^4 \quad (\text{W/m}^2) \quad (22-27)$$

Therefore, the *temperature* of a reradiating surface under steady conditions can easily be determined from the equation above once its radiosity is known. Note that the temperature of a reradiating surface is *independent of its emissivity*. In radiation analysis, the surface resistance of a reradiating surface is disregarded since there is no net heat transfer through it. (This is like the fact that there is no need to consider a resistance in an electrical network if no current is flowing through it.)



**FIGURE 22-21**  
Electrical analogy of surface resistance to radiation.





## Net Radiation Heat Transfer between Any Two Surfaces

Consider two diffuse, gray, and opaque surfaces of arbitrary shape maintained at uniform temperatures, as shown in Fig. 22–22. Recognizing that the radiosity  $J$  represents the rate of radiation leaving a surface per unit surface area and that the view factor  $F_{i \rightarrow j}$  represents the fraction of radiation leaving surface  $i$  that strikes surface  $j$ , the *net* rate of radiation heat transfer from surface  $i$  to surface  $j$  can be expressed as

$$\begin{aligned} \dot{Q}_{i \rightarrow j} &= \left( \begin{array}{l} \text{Radiation leaving} \\ \text{the entire surface } i \\ \text{that strikes surface } j \end{array} \right) - \left( \begin{array}{l} \text{Radiation leaving} \\ \text{the entire surface } j \\ \text{that strikes surface } i \end{array} \right) & (22-28) \\ &= A_i J_i F_{i \rightarrow j} - A_j J_j F_{j \rightarrow i} \quad (\text{W}) \end{aligned}$$

Applying the reciprocity relation  $A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i}$  yields

$$\dot{Q}_{i \rightarrow j} = A_i F_{i \rightarrow j} (J_i - J_j) \quad (\text{W}) \quad (22-29)$$

Again in analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_{i \rightarrow j} = \frac{J_i - J_j}{R_{i \rightarrow j}} \quad (\text{W}) \quad (22-30)$$

where

$$R_{i \rightarrow j} = \frac{1}{A_i F_{i \rightarrow j}} \quad (22-31)$$

is the **space resistance** to radiation. Again the quantity  $J_i - J_j$  corresponds to a *potential difference*, and the net rate of heat transfer between two surfaces corresponds to *current* in the electrical analogy, as illustrated in Fig. 22–22.

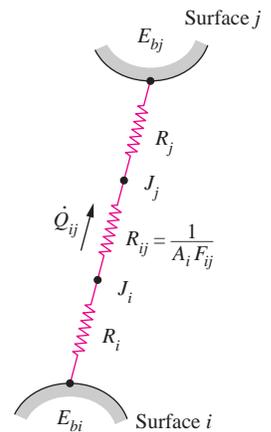
The direction of the net radiation heat transfer between two surfaces depends on the relative magnitudes of  $J_i$  and  $J_j$ . A positive value for  $\dot{Q}_{i \rightarrow j}$  indicates that net heat transfer is *from* surface  $i$  to surface  $j$ . A negative value indicates the opposite.

In an  $N$ -surface enclosure, the conservation of energy principle requires that the net heat transfer from surface  $i$  be equal to the sum of the net heat transfers from surface  $i$  to each of the  $N$  surfaces of the enclosure. That is,

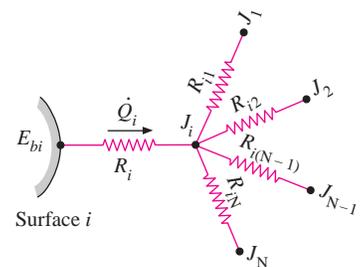
$$\dot{Q}_i = \sum_{j=1}^N \dot{Q}_{i \rightarrow j} = \sum_{j=1}^N A_i F_{i \rightarrow j} (J_i - J_j) = \sum_{j=1}^N \frac{J_i - J_j}{R_{i \rightarrow j}} \quad (\text{W}) \quad (22-32)$$

The network representation of net radiation heat transfer from surface  $i$  to the remaining surfaces of an  $N$ -surface enclosure is given in Fig. 22–23. Note that  $\dot{Q}_{i \rightarrow i}$  (the net rate of heat transfer from a surface to itself) is zero regardless of the shape of the surface. Combining Eqs. 22–25 and 22–32 gives

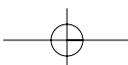
$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^N \frac{J_i - J_j}{R_{i \rightarrow j}} \quad (\text{W}) \quad (22-33)$$



**FIGURE 22–22**  
Electrical analogy of space resistance to radiation.



**FIGURE 22–23**  
Network representation of net radiation heat transfer from surface  $i$  to the remaining surfaces of an  $N$ -surface enclosure.



which has the electrical analogy interpretation that *the net radiation flow from a surface through its surface resistance is equal to the sum of the radiation flows from that surface to all other surfaces through the corresponding space resistances.*

## Methods of Solving Radiation Problems

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates. There are two methods commonly used to solve radiation problems. In the first method, Eqs. 22–32 (for surfaces with specified heat transfer rates) and 22–33 (for surfaces with specified temperatures) are simplified and rearranged as

$$\text{Surfaces with specified net heat transfer rate } \dot{Q}_i \quad \dot{Q}_i = A_i \sum_{j=1}^N F_{i \rightarrow j} (J_i - J_j) \quad (22-34)$$

$$\text{Surfaces with specified temperature } T_i \quad \sigma T_i^4 = J_i + \frac{1 - \varepsilon_i}{\varepsilon_i} \sum_{j=1}^N F_{i \rightarrow j} (J_i - J_j) \quad (22-35)$$

Note that  $\dot{Q}_i = 0$  for insulated (or reradiating) surfaces, and  $\sigma T_i^4 = J_i$  for black surfaces since  $\varepsilon_i = 1$  in that case. Also, the term corresponding to  $j = i$  will drop out from either relation since  $J_i - J_j = J_i - J_i = 0$  in that case.

The equations above give  $N$  linear algebraic equations for the determination of the  $N$  unknown radiosities for an  $N$ -surface enclosure. Once the radiosities  $J_1, J_2, \dots, J_N$  are available, the unknown heat transfer rates can be determined from Eq. 22–34 while the unknown surface temperatures can be determined from Eq. 22–35. The temperatures of insulated or reradiating surfaces can be determined from  $\sigma T_i^4 = J_i$ . A positive value for  $\dot{Q}_i$  indicates net radiation heat transfer *from* surface  $i$  to other surfaces in the enclosure while a negative value indicates net radiation heat transfer *to* the surface.

The systematic approach described above for solving radiation heat transfer problems is very suitable for use with today's popular equation solvers such as EES, Mathcad, and Matlab, especially when there are a large number of surfaces, and is known as the **direct method** (formerly, the *matrix method*, since it resulted in matrices and the solution required a knowledge of linear algebra). The second method described below, called the **network method**, is based on the electrical network analogy.

The network method was first introduced by A. K. Oppenheim in the 1950s and found widespread acceptance because of its simplicity and emphasis on the physics of the problem. The application of the method is straightforward: draw a surface resistance associated with each surface of an enclosure and connect them with space resistances. Then solve the radiation problem by treating it as an electrical network problem where the radiation heat transfer replaces the current and radiosity replaces the potential.

The network method is not practical for enclosures with more than three or four surfaces, however, because of the increased complexity of the network. Next we apply the method to solve radiation problems in two- and three-surface enclosures.