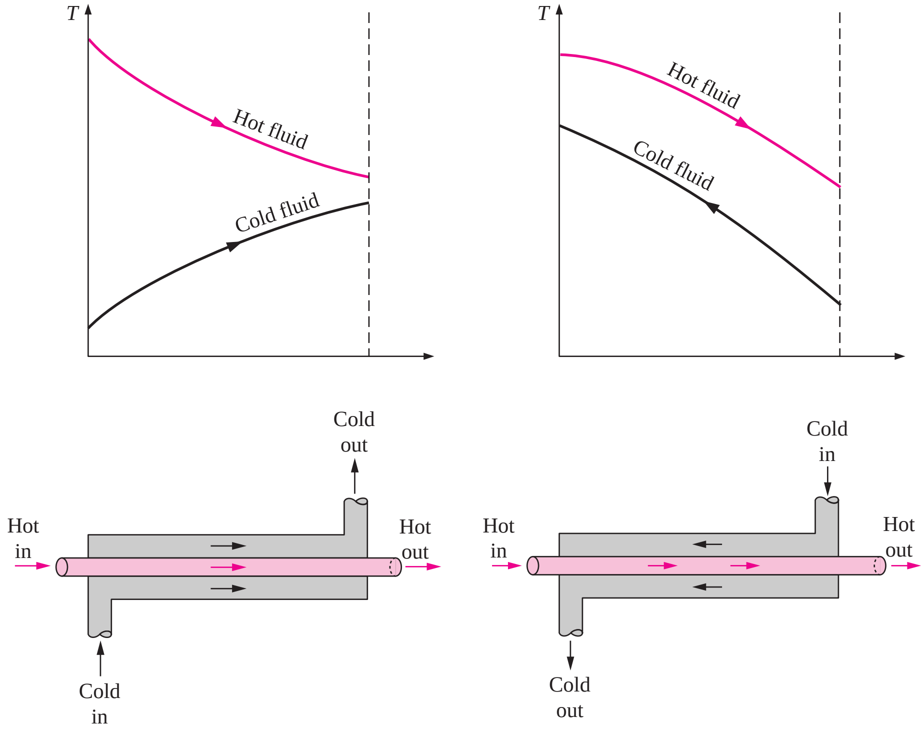
23–1 ▪ TYPES OF HEAT EXCHANGERS

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment. The attempt to match the heat transfer hardware to the heat transfer requirements within the specified constraints has resulted in numerous types of innovative heat exchanger designs.

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Fig. 23–1, called the **double-pipe** heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow,** both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow,** on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact** heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the *area density* β. A heat exchanger with $\beta > 700\ m^2/m^3$ (or 200 ft^2/ft^3) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta \approx 1000\ m^2/m^3$), glass ceramic gas turbine heat exchangers ($\beta \approx 6000\ m^2/m^3$), the regenerator of a Stirling engine ($\beta \approx 15{,}000\ m^2/m^3$), and the human lung ($\beta \approx 20{,}000\ m^2/m^3$). Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in



(*a*) Parallel flow (*b*) Counter flow

FIGURE 23–1
Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.

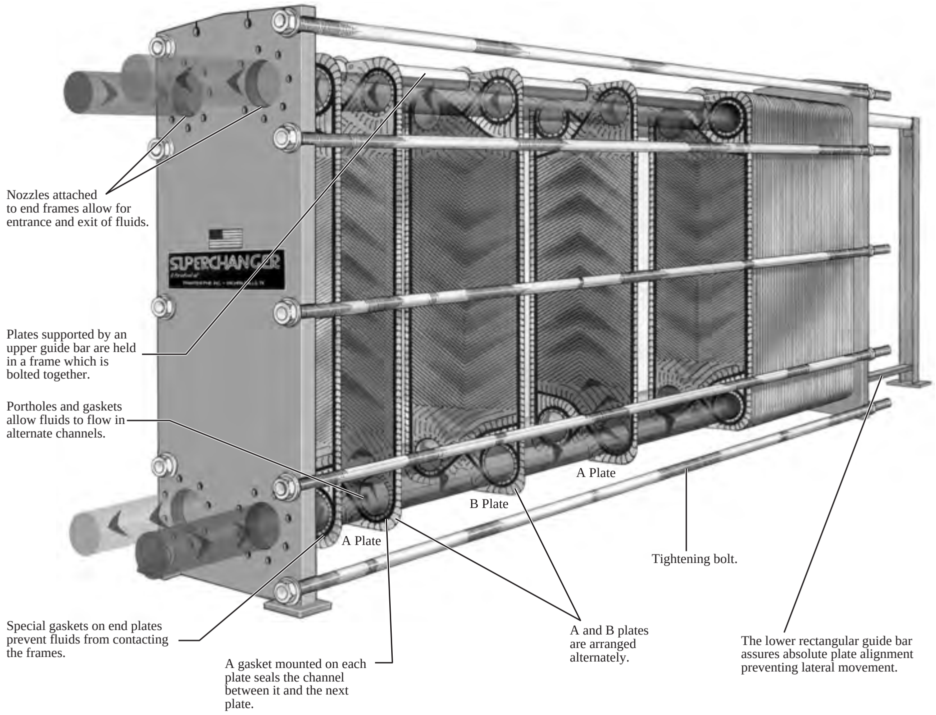


FIGURE 23–6
A plate-and-frame liquid-to-liquid heat exchanger (courtesy of Tranter PHE, Inc.).

The *dynamic*-type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, rejecting this stored heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a *condenser* is a heat exchanger in which one of the fluids is cooled and condenses as it flows through the heat exchanger. A *boiler* is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A *space radiator* is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

23–2 ▪ THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection*, through the wall by *conduction*, and from the wall to the cold fluid again by *convection.* Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Fig. 23–7. Here the subscripts *i* and *o* represent the inner and outer surfaces of the inner

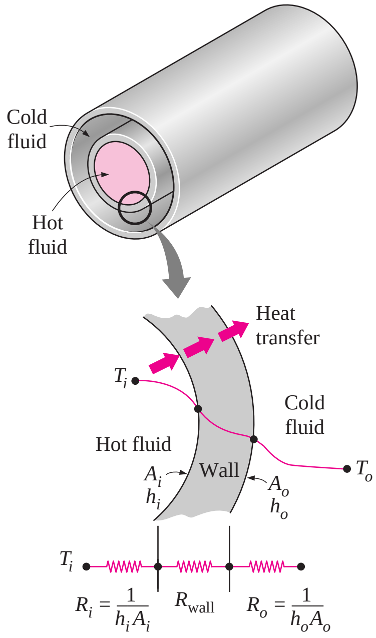


FIGURE 23–7
Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the *thermal resistance* of the tube wall in this case is

$$R_{\text{wall}} = \frac{\ln (D_o/D_i)}{2\pi k L} \tag{23–1}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o} \tag{23–2}$$

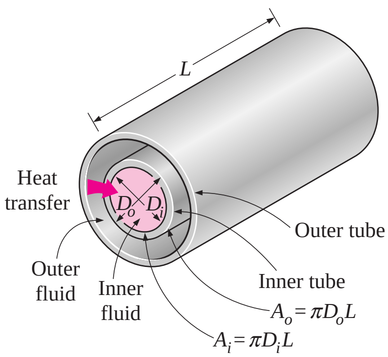


FIGURE 23–8
The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes, $D_i \approx D_o$ and thus $A_i \approx A_o$).

The A_i is the area of the *inner surface* of the wall that separates the two fluids, and A_o is the area of the outer surface of the wall. In other words, A_i and A_o are surface areas of the separating wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$ (Fig. 23–8).

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$\dot{Q} = \frac{\Delta T}{R} = UA\,\Delta T = U_i A_i\,\Delta T = U_o A_o\,\Delta T \tag{23–3}$$

where U is the **overall heat transfer coefficient,** whose unit is W/m$^2 \cdot$ °C, which is identical to the unit of the ordinary convection coefficient h. Canceling ΔT, Eq. 23–3 reduces to

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o} \tag{23–4}$$

Perhaps you are wondering why we have two overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas A_i and A_o, which, in general, are not equal to each other.

Note that $U_i A_i = U_o A_o$, but $U_i \neq U_o$ unless $A_i = A_o$. Therefore, the overall heat transfer coefficient U of a heat exchanger is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible ($R_{\text{wall}} \approx 0$) and the inner and outer surfaces of the tube are almost identical ($A_i \approx A_o \approx A_s$). Then Eq. 23–4 for the overall heat transfer coefficient simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \tag{23–5}$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, h_i and h_o, are determined using the convection relations discussed in earlier chapters.

The overall heat transfer coefficient U in Eq. 23–5 is dominated by the *smaller* convection coefficient, since the inverse of a large number is small. When one of the convection coefficients is *much smaller* than the other (say, $h_i \ll h_o$), we have $1/h_i \gg 1/h_o$, and thus $U \approx h_i$. Therefore, the smaller heat transfer coefficient creates a *bottleneck* on the path of heat flow and seriously impedes heat transfer. This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product UA_s and thus the heat transfer on that side.

Representative values of the overall heat transfer coefficient U are given in Table 23–1. Note that the overall heat transfer coefficient ranges from about 10 W/m² · °C for gas-to-gas heat exchangers to about 10,000 W/m² · °C for heat exchangers that involve phase changes. This is not surprising, since gases have very low thermal conductivities, and phase-change processes involve very high heat transfer coefficients.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes

$$A_s = A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}} \tag{23–6}$$

where A_{fin} is the surface area of the fins and A_{unfinned} is the area of the unfinned portion of the tube surface. For short fins of high thermal conductivity, we can use this total area in the convection resistance relation $R_{\text{conv}} = 1/hA_s$ since the fins in this case will be very nearly isothermal. Otherwise, we should determine the effective surface area A from

$$A_s = A_{\text{unfinned}} + \eta_{\text{fin}} A_{\text{fin}} \tag{23–7}$$

TABLE 23–1

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m² · °C*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60†
	400–850†
Steam-to-air in finned tubes (steam in tubes)	30–300†
	400–4000‡

*Multiply the listed values by 0.176 to convert them to Btu/h · ft² · °F.

†Based on air-side surface area.

‡Based on water- or steam-side surface area.

where η_{fin} is the fin efficiency. This way, the temperature drop along the fins is accounted for. Note that $\eta_{\text{fin}} = 1$ for isothermal fins, and thus Eq. 23–7 reduces to Eq. 23–6 in that case.

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor** R_f, which is a measure of the *thermal resistance* introduced by fouling.

The most common type of fouling is the *precipitation* of solid deposits in a fluid on the heat transfer surfaces. You can observe this type of fouling even in your house. If you check the inner surfaces of your teapot after prolonged use, you will probably notice a layer of calcium-based deposits on the surfaces at which boiling occurs. This is especially the case in areas where the water is hard. The scales of such deposits come off by scratching, and the surfaces can be cleaned of such deposits by chemical treatment. Now imagine those mineral deposits forming on the inner surfaces of fine tubes in a heat exchanger (Fig. 23–9) and the detrimental effect it may have on the flow passage area and the heat transfer. To avoid this potential problem, water in power and process plants is extensively treated and its solid contents are removed before it is allowed to circulate through the system. The solid ash particles in the flue gases accumulating on the surfaces of air preheaters create similar problems.

Another form of fouling, which is common in the chemical process industry, is *corrosion* and other *chemical fouling.* In this case, the surfaces are fouled by the accumulation of the products of chemical reactions on the surfaces. This form of fouling can be avoided by coating metal pipes with glass or using plastic pipes instead of metal ones. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called *biological fouling* and can be prevented by chemical treatment.

In applications where it is likely to occur, fouling should be considered in the design and selection of heat exchangers. In such applications, it may be

FIGURE 23–9
Precipitation fouling of ash particles on superheater tubes. (From *Steam: Its Generation, and Use,* Babcock and Wilcox Co., 1978. Reprinted by permission.)

necessary to select a larger and thus more expensive heat exchanger to ensure that it meets the design heat transfer requirements even after fouling occurs. The periodic cleaning of heat exchangers and the resulting down time are additional penalties associated with fouling.

The fouling factor is obviously zero for a new heat exchanger and increases with time as the solid deposits build up on the heat exchanger surface. The fouling factor depends on the *operating temperature* and the *velocity* of the fluids, as well as the length of service. Fouling increases with *increasing temperature* and *decreasing velocity.*

The overall heat transfer coefficient relation given above is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For an unfinned shell-and-tube heat exchanger, it can be expressed as

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o} \quad \textbf{(23–8)}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

Representative values of fouling factors are given in Table 23–2. More comprehensive tables of fouling factors are available in handbooks. As you would expect, considerable uncertainty exists in these values, and they should be used as a guide in the selection and evaluation of heat exchangers to account for the effects of anticipated fouling on heat transfer. Note that most fouling factors in the table are of the order of 10^{-4} m$^2 \cdot$ °C/W, which is equivalent to the thermal resistance of a 0.2-mm-thick limestone layer ($k = 2.9$ W/m · °C) per unit surface area. Therefore, in the absence of specific data, we can assume the surfaces to be coated with 0.2 mm of limestone as a starting point to account for the effects of fouling.

TABLE 23–2

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

(*Source:* Tubular Exchange Manufacturers Association.)

Fluid	R_f, m^2 · °C/W
Distilled water, seawater, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

EXAMPLE 23–1 Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

SOLUTION Hot oil is cooled by water in a double-tube counter-flow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the oil and water flow are fully developed. **3** Properties of the oil and water are constant.

Properties The properties of water at 45°C are (Table A–15)

$$\rho = 990 \text{ kg/m}^3 \qquad \text{Pr} = 3.91$$
$$k = 0.637 \text{ W/m} \cdot \text{°C} \qquad \upsilon = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2\text{/s}$$

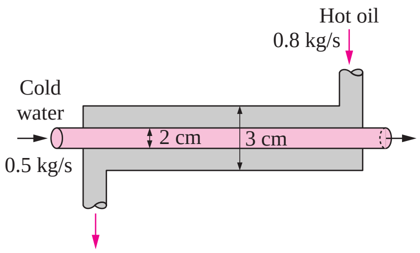


FIGURE 23–10
Schematic for Example 23–1.

The properties of oil at 80°C are (Table A–19).

$$\rho = 852 \text{ kg/m}^3 \qquad \text{Pr} = 490$$
$$k = 0.138 \text{ W/m} \cdot \text{°C} \qquad \upsilon = 37.5 \times 10^{-6} \text{ m}^2\text{/s}$$

Analysis The schematic of the heat exchanger is given in Fig. 23–10. The overall heat transfer coefficient U can be determined from Eq. 23–5:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The mean velocity of water in the tube and the Reynolds number are

$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\upsilon} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2\text{/s}} = 53{,}490$$

which is greater than 4000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8}\text{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

Then,

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.637 \text{ W/m} \cdot \text{°C}}{0.02 \text{ m}}(240.6) = 7663 \text{ W/m}^2 \cdot \text{°C}$$

Now we repeat the analysis above for oil. The properties of oil at 80°C are

$$\rho = 852 \text{ kg/m}^3 \qquad \upsilon = 37.5 \times 10^{-6} \text{ m}^2\text{/s}$$
$$k = 0.138 \text{ W/m} \cdot \text{°C} \qquad \text{Pr} = 490$$

The hydraulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01 \text{ m}$$

The mean velocity and the Reynolds number in this case are

$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8 \text{ kg/s}}{(852 \text{ kg/m}^3)[\frac{1}{4}\pi(0.03^2 - 0.02^2)] \text{ m}^2} = 2.39 \text{ m/s}$$

and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\upsilon} = \frac{(2.39 \text{ m/s})(0.01 \text{ m})}{37.5 \times 10^{-6} \text{ m}^2\text{/s}} = 637$$

which is less than 4000. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 23–3 by interpolation to be

$$\text{Nu} = 5.45$$

TABLE 23–3

Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal (Kays and Perkins)

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

and

$$h_o = \frac{k}{D_h}\,\text{Nu} = \frac{0.138 \text{ W/m}\cdot°\text{C}}{0.01 \text{ m}}(5.45) = 75.2 \text{ W/m}^2\cdot°\text{C}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{7663 \text{ W/m}^2\cdot°\text{C}} + \dfrac{1}{75.2 \text{ W/m}^2\cdot°\text{C}}} = \mathbf{74.5\ W/m^2\cdot°C}$$

Discussion Note that $U \approx h_o$ in this case, since $h_i \gg h_o$. This confirms our earlier statement that the overall heat transfer coefficient in a heat exchanger is dominated by the smaller heat transfer coefficient when the difference between the two values is large.

To improve the overall heat transfer coefficient and thus the heat transfer in this heat exchanger, we must use some enhancement techniques on the oil side, such as a finned surface.

EXAMPLE 23–2 Effect of Fouling on the Overall Heat Transfer Coefficient

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (k = 15.1 W/m · °C) inner tube of inner diameter D_i = 1.5 cm and outer diameter D_o = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be h_i = 800 W/m^2 · °C on the inner surface of the tube and h_o = 1200 W/m^2 · °C on the outer surface. For a fouling factor of $R_{f,i}$ = 0.0004 m^2 · °C/W on the tube side and $R_{f,o}$ = 0.0001 m^2 · °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

SOLUTION The heat transfer coefficients and the fouling factors on the tube and shell sides of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The schematic of the heat exchanger is given in Fig. 23–11. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Eq. 23–8 as

$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

Substituting, the total thermal resistance is determined to be

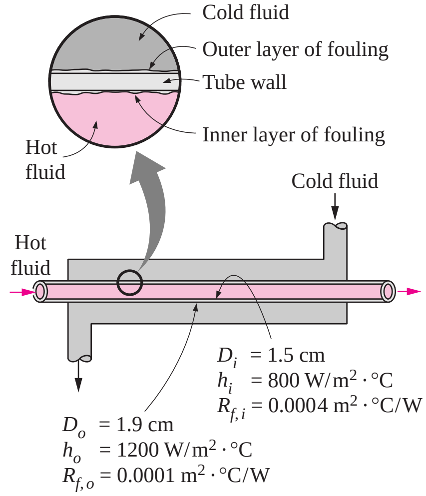


FIGURE 23–11
Schematic for Example 23–2.

$$R = \frac{1}{(800 \text{ W/m}^2 \cdot °\text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2 \cdot °\text{C/W}}{0.0471 \text{ m}^2} + \frac{\ln(0.019/0.015)}{2\pi(15.1 \text{ W/m} \cdot °\text{C})(1 \text{ m})} + \frac{0.0001 \text{ m}^2 \cdot °\text{C/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2 \cdot °\text{C})(0.0597 \text{ m}^2)}$$

$$= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)°\text{C/W}$$

$$= \mathbf{0.0532°C/W}$$

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficients based on the inner and outer surfaces of the tube are determined again from Eq. 23–8 to be

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532 \text{ °C/W})(0.0471 \text{ m}^2)} = \mathbf{399 \text{ W/m}^2 \cdot °C}$$

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532 \text{ °C/W})(0.0597 \text{ m}^2)} = \mathbf{315 \text{ W/m}^2 \cdot °C}$$

Discussion Note that the two overall heat transfer coefficients differ significantly (by 27 percent) in this case because of the considerable difference between the heat transfer surface areas on the inner and the outer sides of the tube. For tubes of negligible thickness, the difference between the two overall heat transfer coefficients would be negligible.

23–3 ▪ ANALYSIS OF HEAT EXCHANGERS

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to *select a heat exchanger* that will achieve a *specified temperature change* in a fluid stream of known mass flow rate, or to *predict the outlet temperatures* of the hot and cold fluid streams in a *specified heat exchanger.*

In upcoming sections, we will discuss the two methods used in the analysis of heat exchangers. Of these, the *log mean temperature difference* (or LMTD) method is best suited for the first task and the *effectiveness–NTU* method for the second task as just stated. But first we present some general considerations.

Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modeled as *steady-flow* devices. As such, the mass flow rate of each fluid remains constant, and the fluid properties such as temperature and velocity at any inlet or outlet remain the same. Also, the fluid streams experience little or no change in their velocities and elevations, and thus the *kinetic* and *potential energy changes* are negligible. The *specific heat* of a fluid, in general, changes with temperature. But, in a specified temperature range, it can be treated as a constant at some average value with little loss in accuracy. *Axial heat conduction* along the tube is usually insignificant and can be considered negligible. Finally, the outer surface of the heat exchanger is assumed to be *perfectly insulated,* so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only.

The idealizations stated above are closely approximated in practice, and they greatly simplify the analysis of a heat exchanger with little sacrifice of accuracy. Therefore, they are commonly used. Under these assumptions, the *first law of thermodynamics* requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \tag{23–9}$$

and

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,\,\text{in}} - T_{h,\,\text{out}}) \tag{23–10}$$

where the subscripts *c* and *h* stand for *cold* and *hot* fluids, respectively, and

$$\begin{aligned} \dot{m}_c,\, \dot{m}_h &= \text{mass flow rates} \\ C_{pc},\, C_{ph} &= \text{specific heats} \\ T_{c,\,\text{out}},\, T_{h,\,\text{out}} &= \text{outlet temperatures} \\ T_{c,\,\text{in}},\, T_{h,\,\text{in}} &= \text{inlet temperatures} \end{aligned}$$

Note that the heat transfer rate $\dot{Q}$ is taken to be a positive quantity, and its direction is understood to be from the hot fluid to the cold one in accordance with the second law of thermodynamics.

In heat exchanger analysis, it is often convenient to combine the product of the *mass flow rate* and the *specific heat* of a fluid into a single quantity. This quantity is called the **heat capacity rate** and is defined for the hot and cold fluid streams as

$$C_h = \dot{m}_h C_{ph} \qquad \text{and} \qquad C_c = \dot{m}_c C_{pc} \tag{23–11}$$

The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1°C as it flows through a heat exchanger. Note that in a heat exchanger, the fluid with a *large* heat capacity rate will experience a *small* temperature change, and the fluid with a *small* heat capacity rate will experience a *large* temperature change. Therefore, *doubling* the mass flow rate of a fluid while leaving everything else unchanged will *halve* the temperature change of that fluid.

With the definition of the heat capacity rate above, Eqs. 23–9 and 23–10 can also be expressed as

$$\dot{Q} = C_c(T_{c,\,\text{out}} - T_{c,\,\text{in}}) \tag{23–12}$$

and

$$\dot{Q} = C_h(T_{h,\,\text{in}} - T_{h,\,\text{out}}) \tag{23–13}$$

That is, the heat transfer rate in a heat exchanger is equal to the heat capacity rate of either fluid multiplied by the temperature change of that fluid. Note that *the only time the temperature rise of a cold fluid is equal to the temperature drop of the hot fluid is when the heat capacity rates of the two fluids are equal to each other* (Fig. 23–12).

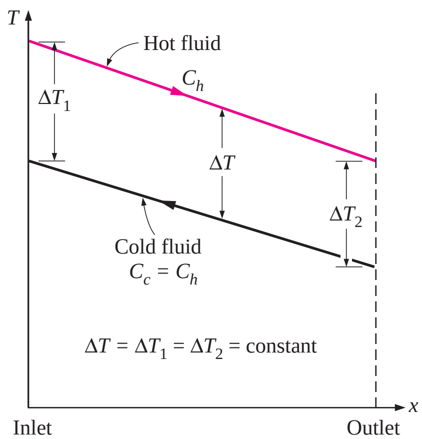


FIGURE 23–12
Two fluids that have the same mass flow rate and the same specific heat experience the same temperature change in a well-insulated heat exchanger.

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers*. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

$$\dot{Q} = \dot{m}h_{fg} \qquad \textbf{(23–14)}$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

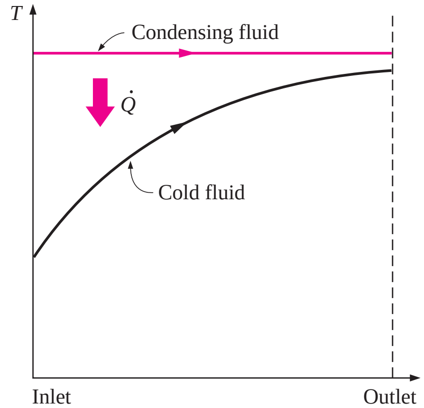
An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in Fig. 23–13. The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is, $C = \dot{m}C_p \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m}C_p\,\Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity*.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

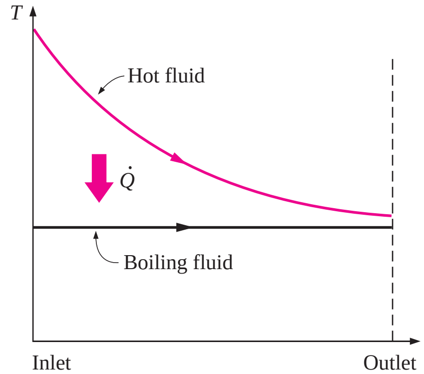
$$\dot{Q} = UA_s\,\Delta T_m \qquad \textbf{(23–15)}$$

where U is the overall heat transfer coefficient, A_s is the heat transfer area, and ΔT_m is an appropriate average temperature difference between the two fluids. Here the surface area A_s can be determined precisely using the dimensions of the heat exchanger. However, the overall heat transfer coefficient U and the temperature difference ΔT between the hot and cold fluids, in general, are not constant and vary along the heat exchanger.

The average value of the overall heat transfer coefficient can be determined as described in the preceding section by using the average convection coefficients for each fluid. It turns out that the appropriate form of the mean temperature difference between the two fluids is *logarithmic* in nature, and its determination is presented in Section 23–4.



(a) Condenser ($C_h \to \infty$)



(b) Boiler ($C_c \to \infty$)

FIGURE 23–13
Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

23–4 ▪ THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a *mean temperature difference* ΔT_m for use in the relation $\dot{Q} = UA_s\,\Delta T_m$.

In order to develop a relation for the equivalent average temperature difference between the two fluids, consider the *parallel-flow double-pipe* heat exchanger shown in Fig. 23–14. Note that the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. As you would expect, the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is.

Assuming the outer surface of the heat exchanger to be well insulated so that any heat transfer occurs between the two fluids, and disregarding any changes in kinetic and potential energy, an energy balance on each fluid in a differential section of the heat exchanger can be expressed as

$$\delta \dot{Q} = -\dot{m}_h C_{ph}\, dT_h \qquad \textbf{(23–16)}$$

and

$$\delta \dot{Q} = \dot{m}_c C_{pc}\, dT_c \qquad \textbf{(23–17)}$$

That is, the rate of heat loss from the hot fluid at any section of a heat exchanger is equal to the rate of heat gain by the cold fluid in that section. The temperature change of the hot fluid is a *negative* quantity, and so a *negative sign* is added to Eq. 23–16 to make the heat transfer rate $\dot{Q}$ a positive quantity. Solving the equations above for dT_h and dT_c gives

$$dT_h = -\frac{\delta \dot{Q}}{\dot{m}_h C_{ph}} \qquad \textbf{(23–18)}$$

and

$$dT_c = \frac{\delta \dot{Q}}{\dot{m}_c C_{pc}} \qquad \textbf{(23–19)}$$

Taking their difference, we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta \dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \qquad \textbf{(23–20)}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as

$$\delta \dot{Q} = U(T_h - T_c)\, dA_s \qquad \textbf{(23–21)}$$

Substituting this equation into Eq. 23–20 and rearranging gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \qquad \textbf{(23–22)}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain

$$\ln \frac{T_{h,\,out} - T_{c,\,out}}{T_{h,\,in} - T_{c,\,in}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \qquad \textbf{(23–23)}$$

Finally, solving Eqs. 23–9 and 23–10 for $\dot{m}_c C_{pc}$ and $\dot{m}_h C_{ph}$ and substituting into Eq. 23–23 gives, after some rearrangement,

$$\dot{Q} = UA_s\, \Delta T_{lm} \qquad \textbf{(23–24)}$$

where

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \qquad \textbf{(23–25)}$$

is the **log mean temperature difference,** which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids

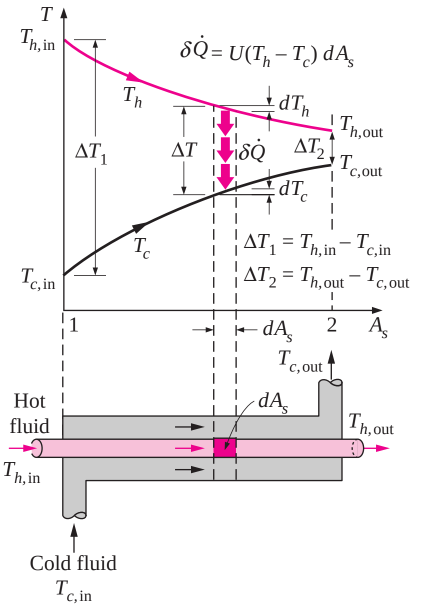
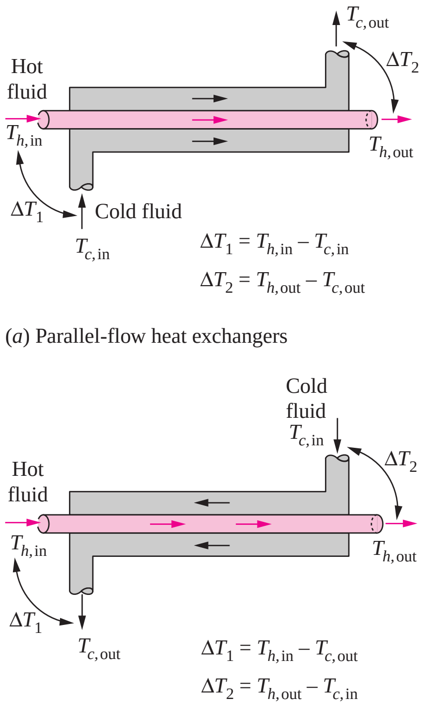


FIGURE 23–14
Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.



(*a*) Parallel-flow heat exchangers

(*b*) Counter-flow heat exchangers

FIGURE 23–15

The ΔT_1 and ΔT_2 expressions in parallel-flow and counter-flow heat exchangers.

at the two ends (inlet and outlet) of the heat exchanger. It makes no difference which end of the heat exchanger is designated as the *inlet* or the *outlet* (Fig. 23–15).

The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet. Thus, it is tempting to use the arithmetic mean temperature $\Delta T_{am} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$ as the average temperature difference. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger and is an *exact* representation of the *average temperature difference* between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference.

Note that ΔT_{lm} is always less than ΔT_{am}. Therefore, using ΔT_{am} in calculations instead of ΔT_{lm} will overestimate the rate of heat transfer in a heat exchanger between the two fluids. When ΔT_1 differs from ΔT_2 by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_1 differs from ΔT_2 by greater amounts. Therefore, we should always use the *logarithmic mean temperature difference* when determining the rate of heat transfer in a heat exchanger.

Counter-Flow Heat Exchangers

The variation of temperatures of hot and cold fluids in a counter-flow heat exchanger is given in Fig. 23–16. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the *cold fluid* in this case may exceed the outlet temperature of the *hot fluid*. In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid. However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid, since this would be a violation of the second law of thermodynamics.

The relation already given for the log mean temperature difference is developed using a parallel-flow heat exchanger, but we can show by repeating the analysis for a counter-flow heat exchanger that is also applicable to counter-flow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in Fig. 23–15.

For specified inlet and outlet temperatures, the log mean temperature difference for a *counter-flow* heat exchanger is always *greater* than that for a parallel-flow heat exchanger. That is, $\Delta T_{lm,\,CF} > \Delta T_{lm,\,PF}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger. Therefore, it is common practice to use counter-flow arrangements in heat exchangers.

In a counter-flow heat exchanger, the temperature difference between the hot and the cold fluids will remain constant along the heat exchanger when the *heat capacity rates* of the two fluids are *equal* (that is, ΔT = constant when $C_h = C_c$ or $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$). Then we have $\Delta T_1 = \Delta T_2$, and the last log mean temperature difference relation gives $\Delta T_{lm} = \frac{0}{0}$, which is indeterminate. It can be shown by the application of l'Hôpital's rule that in this case we have $\Delta T_{lm} = \Delta T_1 = \Delta T_2$, as expected.

A *condenser* or a *boiler* can be considered to be either a parallel- or counter-flow heat exchanger since both approaches give the same result.

23–5 ▪ THE EFFECTIVENESS–NTU METHOD

The log mean temperature difference (LMTD) method discussed in Section 23–4 is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. Once ΔT_{lm}, the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from

$$\dot{Q} = UA_s\,\Delta T_{lm}$$

Therefore, the LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to *select* a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_{lm} and the correction factor F, if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U.
5. Calculate the heat transfer surface area A_s.

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A_s.

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified. The heat transfer surface area A of the heat exchanger in this case is known, but the *outlet temperatures* are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

The LMTD method could still be used for this alternative problem, but the procedure would require tedious iterations, and thus it is not practical. In an attempt to eliminate the iterations from the solution of such problems, Kays and London came up with a method in 1955 called the **effectiveness–NTU method,** which greatly simplified heat exchanger analysis.

This method is based on a dimensionless parameter called the **heat transfer effectiveness** ε, defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \tag{23–29}$$

The *actual* heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) = C_h(T_{h,\,in} - T_{h,\,out}) \tag{23–30}$$

where $C_c = \dot{m}_c C_{pc}$ and $C_h = \dot{m}_c C_{ph}$ are the heat capacity rates of the cold and the hot fluids, respectively.

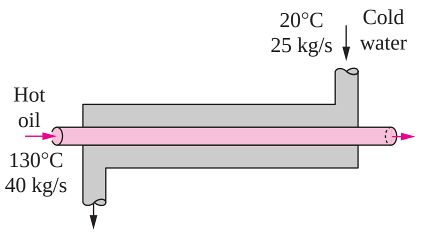
To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is,

$$\Delta T_{\max} = T_{h,\text{ in}} - T_{c,\text{ in}} \tag{23–31}$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the *smaller* heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (Fig. 23–23)

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \tag{23–32}$$

where $C_{\min}$ is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$. This is further clarified by Example 23–7.



$C_c = \dot{m}_c C_{pc} = 104.5$ kW/°C
$C_h = \dot{m}_c C_{ph} = 92$ kW/°C
$C_{\min} = 92$ kW/°C
$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = 110$°C
$\dot{Q}_{\max} = C_{\min}\,\Delta T_{\max} = 10{,}120$ kW

FIGURE 23–23
The determination of the maximum rate of heat transfer in a heat exchanger.

EXAMPLE 23–7 Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot-water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $C_p = 4.18$ kJ/kg · °C, determine the maximum heat transfer rate and the outlet temperatures of the cold- and the hot-water streams for this limiting case.

SOLUTION Cold- and hot-water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heat of water is given to be $C_p = 4.18$ kJ/kg · °C.

Analysis A schematic of the heat exchanger is given in Fig. 23–24. The heat capacity rates of the hot and cold fluids are determined from

$$C_h = \dot{m}_h C_{ph} = (2\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C}) = 8.36\text{ kW/}^\circ\text{C}$$

and

$$C_c = \dot{m}_c C_{pc} = (8\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C}) = 33.4\text{ kW/}^\circ\text{C}$$

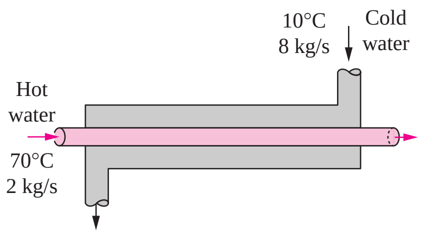


FIGURE 23–24
Schematic for Example 23–7.

Therefore

$$C_{\min} = C_h = 8.36 \text{ kW/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Eq. 23–32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (8.36 \text{ kW/°C})(70 - 10)\text{°C} \\ &= \textbf{502 kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area.

The maximum temperature difference in this heat exchanger is $\Delta T_{\max} = T_{h,\text{ in}} - T_{c,\text{ in}} = (70 - 10)\text{°C} = 60\text{°C}$. Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

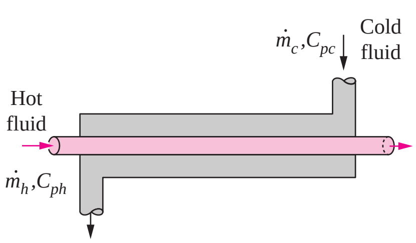
$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \longrightarrow T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c} = 10\text{°C} + \frac{502 \text{ kW}}{33.4 \text{ kW/°C}} = \textbf{25°C}$$

$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \longrightarrow T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h} = 70\text{°C} - \frac{502 \text{ kW}}{8.38 \text{ kW/°C}} = \textbf{10°C}$$

Discussion Note that the hot water is cooled to the limit of 10°C (the inlet temperature of the cold-water stream), but the cold water is heated to 25°C only when maximum heat transfer occurs in the heat exchanger. This is not surprising, since the mass flow rate of the hot water is only one-fourth that of the cold water, and, as a result, the temperature of the cold water increases by 0.25°C for each 1°C drop in the temperature of the hot water.

You may be tempted to think that the cold water should be heated to 70°C in the limiting case of maximum heat transfer. But this will require the temperature of the hot water to drop to −170°C (below 10°C), which is impossible. Therefore, heat transfer in a heat exchanger reaches its maximum value when the fluid with the smaller heat capacity rate (or the smaller mass flow rate when both fluids have the same specific heat value) experiences the maximum temperature change. This example explains why we use $C_{\min}$ in the evaluation of $\dot{Q}_{\max}$ instead of $C_{\max}$.

We can show that the hot water will leave at the inlet temperature of the cold water and vice versa in the limiting case of maximum heat transfer when the mass flow rates of the hot- and cold-water streams are identical (Fig. 23–25). We can also show that the outlet temperature of the cold water will reach the 70°C limit when the mass flow rate of the hot water is greater than that of the cold water.



$$\begin{aligned}\dot{Q} &= \dot{m}_h C_{ph}\, \Delta T_h \\ &= \dot{m}_c C_{pc}\, \Delta T_c\end{aligned}$$

If $\quad \dot{m}_c C_{pc} = \dot{m}_h C_{ph}$

then $\quad \Delta T_h = \Delta T_c$

FIGURE 23–25
The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the mass flow rates and the specific heats of the hot and cold fluids are identical.

The determination of $\dot{Q}_{\max}$ requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates*, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \qquad \textbf{(23–33)}$$

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group UA_s/C_{min}. This quantity is called the **number of transfer units NTU** and is expressed as

$$\text{NTU} = \frac{UA_s}{C_{min}} = \frac{UA_s}{(\dot{m}C_p)_{min}} \tag{23–39}$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s. Therefore, for specified values of U and C_{min}, the value of NTU *is a measure of the heat transfer surface area* A_s. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the **capacity ratio *c*** as

$$c = \frac{C_{min}}{C_{max}} \tag{23–40}$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c. That is,

$$\varepsilon = \text{function}\,(UA_s/C_{min}, C_{min}/C_{max}) = \text{function}\,(\text{NTU}, c)$$

Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table 23–4. The effectivenesses of some common types of heat exchangers are also plotted in Fig. 23–26. More

TABLE 23–4

Effectiveness relations for heat exchangers: $\text{NTU} = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp\,[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp\,[-\text{NTU}(1 - c)]}{1 - c\exp\,[-\text{NTU}(1 - c)]}$
2 *Shell-and-tube:* One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp\,[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp\,[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow* (*single-pass*)	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}\,[\exp\,(-c\ \text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\,\{1 - c[1 - \exp\,(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp\,(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with* $c = 0$	$\varepsilon = 1 - \exp(-\text{NTU})$

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