Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\,\text{Nu} = \frac{0.02551 \text{ W/m} \cdot {}^\circ\text{C}}{0.25 \text{ m}}(135) = \mathbf{13.8 \text{ W/m}^2 \cdot {}^\circ\text{C}}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25 \text{ m})^2 = 0.1963 \text{ m}^2$$
$$\dot{Q}_{\text{ave}} = hA_s(T_{s,\text{ ave}} - T_\infty) = (13.8 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.1963 \text{ m}^2)(250 - 25)^\circ\text{C} = 610 \text{ W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho \tfrac{1}{6}\pi D^3 = (8055 \text{ kg/m}^3)\, \tfrac{1}{6}\pi(0.25 \text{ m})^3 = 65.9 \text{ kg}$$
$$Q_{\text{total}} = mC_p(T_2 - T_1) = (65.9 \text{ kg})(480 \text{ J/kg} \cdot {}^\circ\text{C})(300 - 200)^\circ\text{C} = 3{,}163{,}000 \text{ J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{\text{ave}}} = \frac{3{,}163{,}000 \text{ J}}{610 \text{ J/s}} = 5185 \text{ s} = \mathbf{1 \text{ h } 26 \text{ min}}$$

Discussion The time of cooling could also be determined more accurately using the transient temperature charts or relations introduced in Chap. 18. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between 1 and 2 h.

19–5 ▪ GENERAL CONSIDERATIONS FOR PIPE FLOW

Liquid or gas flow through pipes or ducts is commonly used in practice in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a conduit that is sufficiently long to accomplish the desired heat transfer.

The general aspects of flow in pipes were considered in Chap. 14. The Reynolds number for flow through a pipe of inside diameter D was defined as

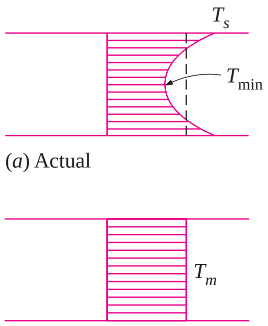
$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\upsilon} \qquad \textbf{(19–31)}$$

where $\mathcal{V}_m$ is the mean velocity and $\upsilon = \mu/\rho$ is the kinematic viscosity of the fluid. Under most conditions, the flow in a pipe is said to be laminar for $\text{Re} < 2300$, turbulent for $\text{Re} > 4000$, and transitional in between.

When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center. In fluid flow it is convenient to work with an **average** or **mean temperature** T_m that remains uniform at a cross section. Unlike the mean velocity, the mean temperature T_m *will change* in the flow direction whenever the fluid is heated or cooled.

The value of the mean temperature T_m is determined from the requirement that the *conservation of energy* principle be satisfied. That is, the energy transported by the fluid through a cross section in actual flow must be equal to the energy that would be transported through the same cross section if the fluid were at a constant temperature T_m. This can be expressed mathematically as (Fig. 19–19)

$$\dot{E}_{\text{fluid}} = \dot{m}C_pT_m = \int_{\dot{m}} C_pT\,\delta\dot{m} = \int_{A_c} \rho C_pT\mathcal{V}\,dA_c \qquad \textbf{(19–32)}$$



(*b*) Idealized

FIGURE 19–19
Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

where C_p is the specific heat of the fluid. Note that the product $\dot{m}C_pT_m$ at any cross section along the tube represents the *energy flow* with the fluid at that cross section. Then the mean temperature of a fluid with constant density and specific heat flowing in a circular pipe of radius R can be expressed as

$$T_m = \frac{\int_{\dot{m}} C_pT\,\delta\dot{m}}{\dot{m}C_p} = \frac{\int_0^R C_pT(\rho\mathcal{V}2\pi r\,dr)}{\rho\mathcal{V}_m(\pi R^2)C_p} = \frac{2}{\mathcal{V}_mR^2}\int_0^R T(r, x)\,\mathcal{V}(r, x)\,r\,dr \qquad \textbf{(19–33)}$$

Note that the mean temperature T_m of a fluid changes during heating or cooling. Also, the fluid properties in internal flow are usually evaluated at the *bulk mean fluid temperature*, which is the arithmetic average of the mean temperatures at the inlet and the exit. That is, $T_b = (T_{m,\,i} + T_{m,\,e})/2$.

Thermal Entrance Region

The development of the velocity boundary layer was discussed in Chap. 14. Now consider a fluid at a uniform temperature entering a circular tube whose surface is maintained at a different temperature. This time, the fluid particles in the layer in contact with the surface of the tube will assume the surface temperature. This will initiate convection heat transfer in the tube and the development of a *thermal boundary layer* along the tube. The thickness of this boundary layer also increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Fig. 19–20.

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the **thermal entrance region,** and the length of this region is called the **thermal entry length** L_t. Flow in the thermal entrance region is called *thermally developing flow* since this is the region where

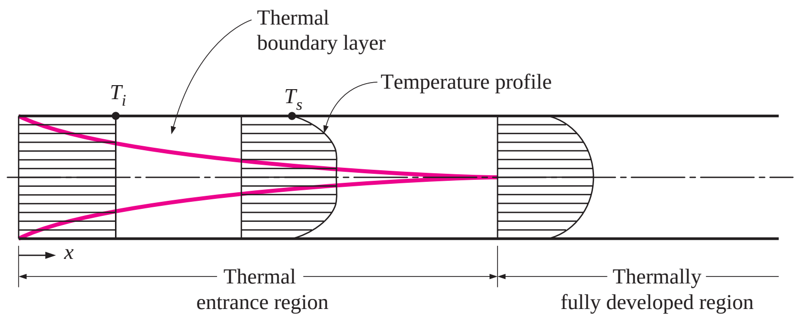


FIGURE 19–20
The development of the thermal boundary layer in a tube. (The fluid in the tube is being cooled.)

the temperature profile develops. The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T)/(T_s - T_m)$ remains unchanged is called the **thermally fully developed region.** The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called *fully developed flow.* That is,

Hydrodynamically fully developed:
$$\frac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r) \tag{19–34}$$

Thermally fully developed:
$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0 \tag{19–35}$$

The friction factor is related to the shear stress at the surface, which is related to the slope of the velocity profile at the surface. Noting that the velocity profile remains unchanged in the hydrodynamically fully developed region, the friction factor also remains constant in that region. A similar argument can be given for the heat transfer coefficient in the thermally fully developed region.

In a thermally fully developed region, the derivative of $(T_s - T)/(T_s - T_m)$ with respect to x is zero by definition, and thus $(T_s - T)/(T_s - T_m)$ is independent of x. Then the derivative of $(T_s - T)/(T_s - T_m)$ with respect r must also be independent of x. That is,

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=R} = \frac{-(\partial T/\partial r)|_{r=R}}{T_s - T_m} \neq f(x) \tag{19–36}$$

Surface heat flux can be expressed as

$$\dot{q}_s = h_x(T_s - T_m) = k\frac{\partial T}{\partial r}\bigg|_{r=R} \longrightarrow h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m} \tag{19–37}$$

which, from Eq. 19–36, is independent of x. Thus we conclude that *in the thermally fully developed region of a tube, the local convection coefficient is constant* (does not vary with x). Therefore, both *the friction and convection coefficients remain constant in the fully developed region of a tube.*

Note that the *temperature profile* in the thermally fully developed region may vary with x in the flow direction. That is, unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region, and it usually is. However, the dimensionless temperature profile already defined remains unchanged in the thermally developed region when the temperature or heat flux at the tube surface remains constant.

During laminar flow in a tube, the magnitude of the dimensionless Prandtl number Pr is a measure of the relative growth of the velocity and thermal boundary layers. For fluids with $\text{Pr} \approx 1$, such as gases, the two boundary layers essentially coincide with each other. For fluids with $\text{Pr} \gg 1$, such as oils, the velocity boundary layer outgrows the thermal boundary layer. As a result, the hydrodynamic entry length is smaller than the thermal entry length. The opposite is true for fluids with $\text{Pr} \ll 1$ such as liquid metals.

Consider a fluid that is being heated (or cooled) in a tube as it flows through it. The friction factor and the heat transfer coefficient are *highest* at the tube inlet where the thickness of the boundary layers is zero, and decrease

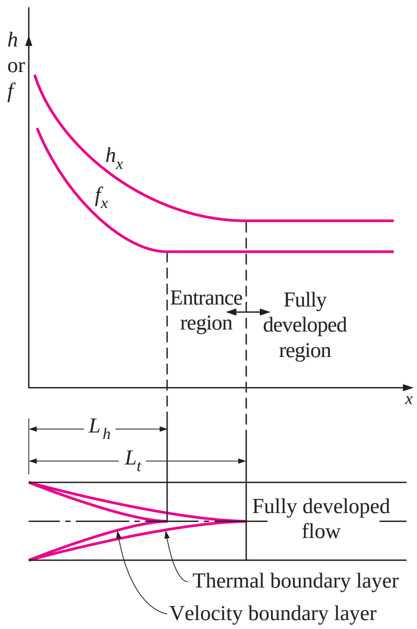


FIGURE 19–21
Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube (Pr > 1).

gradually to the fully developed values, as shown in Fig. 19–21. Therefore, the pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *enhance* the average friction and heat transfer coefficients for the entire tube. This enhancement can be significant for short tubes but negligible for long ones.

In laminar flow, the hydrodynamic and thermal entry lengths are given approximately as [see Kays and Crawford (1993) and Shah and Bhatti (1987)].

$$L_{h,\text{ laminar}} \approx 0.05 \text{ Re } D \tag{19–38}$$

$$L_{t,\text{ laminar}} \approx 0.05 \text{ Re Pr } D = \text{Pr } L_{h,\text{ laminar}} \tag{19–39}$$

The hydrodynamic entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker. It is 11D at Re = 10,000, and increases to 43D at Re = 10^5. In practice, it is generally agreed that the entrance effects are confined within a tube length of 10 diameters, and the hydrodynamic and thermal entry lengths are approximately taken to be

$$L_{h,\text{ turbulent}} \approx L_{t,\text{ turbulent}} \approx 10D \tag{19–40}$$

The variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux is given in Fig. 19–22 for the range of Reynolds numbers encountered in heat transfer equipment. We make these important observations from this figure:

- The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region.
- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for $x > 10D$.
- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions. Therefore, Nusselt number

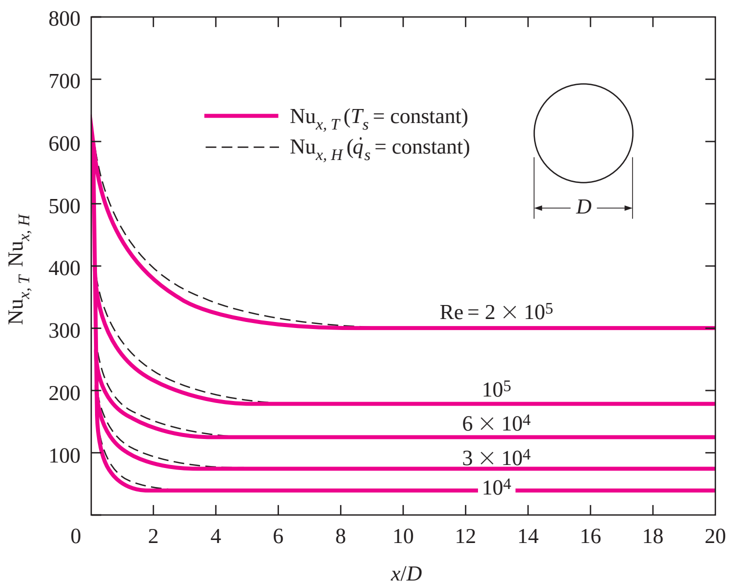


FIGURE 19–22
Variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux [Deissler (1953)].

is insensitive to the type of thermal boundary condition, and the turbulent flow correlations can be used for either type of boundary condition.

Precise correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature. However, the tubes used in practice in forced convection are usually several times the length of either entrance region, and thus the flow through the tubes is often assumed to be fully developed for the entire length of the tube. This simplistic approach gives *reasonable* results for long tubes and *conservative* results for short ones.

19–6 ▪ GENERAL THERMAL ANALYSIS

You will recall that in the absence of any work interactions (such as electric resistance heating), the conservation of energy equation for the steady flow of a fluid in a tube can be expressed as (Fig. 19–23)

$$\dot{Q} = \dot{m}C_p(T_e - T_i) \qquad \text{(W)} \tag{19–41}$$

where T_i and T_e are the mean fluid temperatures at the inlet and exit of the tube, respectively, and $\dot{Q}$ is the rate of heat transfer to or from the fluid. Note that the temperature of a fluid flowing in a tube remains constant in the absence of any energy interactions through the wall of the tube.

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature* (T_s = constant) or *constant surface heat flux* ($\dot{q}_s$ = constant). For example, the constant surface temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube. The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad \text{(W/m}^2\text{)} \tag{19–42}$$

where h_x is the *local* heat transfer coefficient and T_s and T_m are the surface and the mean fluid temperatures at that location. Note that the mean fluid temperature T_m of a fluid flowing in a tube must change during heating or cooling. Therefore, when $h_x = h$ = constant, the surface temperature T_s must change when $\dot{q}_s$ = constant, and the surface heat flux $\dot{q}_s$ must change when T_s = constant. Thus we may have either T_s = constant or $\dot{q}_s$ = constant at the surface of a tube, but not both. Next we consider convection heat transfer for these two common cases.

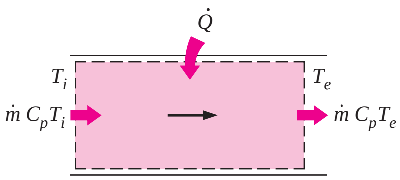


FIGURE 19–23
The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

Constant Surface Heat Flux ($\dot{q}_s$ = constant)

In the case of $\dot{q}_s$ = constant, the rate of heat transfer can also be expressed as

$$\dot{Q} = \dot{q}_sA_s = \dot{m}C_p(T_e - T_i) \qquad \text{(W)} \tag{19–43}$$

Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_sA_s}{\dot{m}C_p} \tag{19–44}$$

Note that the mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction (A_s is equal to the perimeter, which is constant, times the tube length).

The surface temperature in the case of constant surface heat flux $\dot{q}_s$ can be determined from

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h} \qquad \textbf{(19–45)}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant and thus $T_s - T_m =$ constant (Fig. 19–24). Of course this is true when the fluid properties remain constant during flow.

The slope of the mean fluid temperature T_m on a T-x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx shown in Fig. 19–25. It gives

$$\dot{m}C_p\,dT_m = \dot{q}_s(p\,dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}C_p} = \text{constant} \qquad \textbf{(19–46)}$$

where p is the perimeter of the tube.

Noting that both $\dot{q}_s$ and h are constants, the differentiation of Eq. 19–45 with respect to x gives

$$\frac{dT_m}{dx} = \frac{dT_s}{dx} \qquad \textbf{(19–47)}$$

Also, the requirement that the dimensionless temperature profile remains unchanged in the fully developed region gives

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx} \qquad \textbf{(19–48)}$$

since $T_s - T_m =$ constant. Combining Eqs. 19–46, 19–47, and 19–48 gives

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}C_p} = \text{constant} \qquad \textbf{(19–49)}$$

Then we conclude that *in fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube* (Fig. 19–26).

For a circular tube, $p = 2\pi R$ and $\dot{m} = \rho \mathcal{V}_m A_c = \rho \mathcal{V}_m(\pi R^2)$, and Eq. 19–49 becomes

Circular tube:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant} \qquad \textbf{(19–50)}$$

where $\mathcal{V}_m$ is the mean velocity of the fluid.

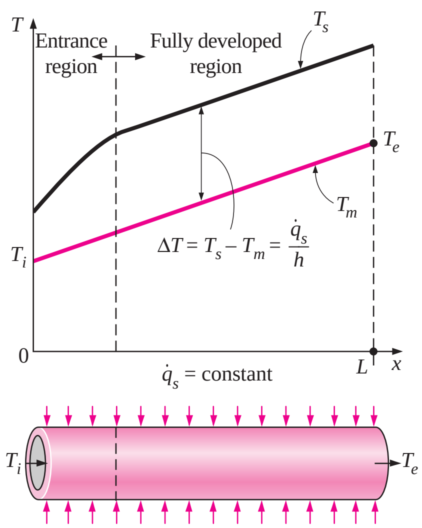


FIGURE 19–24
Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

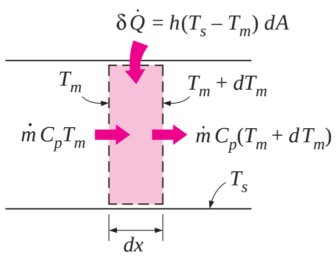


FIGURE 19–25
Energy interactions for a differential control volume in a tube.

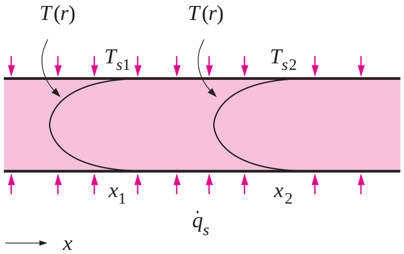


FIGURE 19–26
The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux.

Constant Surface Temperature (T_s = constant)

From Newton's law of cooling, the rate of heat transfer to or from a fluid flowing in a tube can be expressed as

$$\dot{Q} = hA_s\Delta T_{ave} = hA_s(T_s - T_m)_{ave} \qquad \text{(W)} \qquad \textbf{(19–51)}$$

where h is the average convection heat transfer coefficient, A_s is the heat transfer surface area (it is equal to πDL for a circular pipe of length L), and ΔT_{ave} is some appropriate *average* temperature difference between the fluid and the surface. Below we discuss two suitable ways of expressing ΔT_{ave}.

In the constant surface temperature (T_s = constant) case, ΔT_{ave} can be expressed *approximately* by the **arithmetic mean temperature difference** ΔT_{am} as

$$\Delta T_{\text{ave}} \approx \Delta T_{\text{am}} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2}$$
$$= T_s - T_b \qquad \textbf{(19–52)}$$

where $T_b = (T_i + T_e)/2$ is the *bulk mean fluid temperature*, which is the *arithmetic average* of the mean fluid temperatures at the inlet and the exit of the tube.

Note that the *arithmetic mean temperature difference* ΔT_{am} is simply the *average* of the *temperature differences* between the surface and the fluid at the inlet and the exit of the tube. Inherent in this definition is the assumption that the mean fluid temperature varies linearly along the tube, which is hardly ever the case when T_s = constant. This simple approximation often gives acceptable results, but not always. Therefore, we need a better way to evaluate ΔT_{ave}.

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature of T_s. We know that the mean temperature of the fluid T_m will increase in the flow direction as a result of heat transfer. The energy balance on a differential control volume shown in Fig. 19–25 gives

$$\dot{m}C_p\, dT_m = h(T_s - T_m)\, dA_s \qquad \textbf{(19–53)}$$

That is, the increase in the energy of the fluid (represented by an increase in its mean temperature by dT_m) is equal to the heat transferred to the fluid from the tube surface by convection. Noting that the differential surface area is $dA_s = pdx$, where p is the perimeter of the tube, and that $dT_m = -d(T_s - T_m)$, since T_s is constant, the last relation can be rearranged as

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m}C_p}\, dx \qquad \textbf{(19–54)}$$

Integrating from $x = 0$ (tube inlet where $T_m = T_i$) to $x = L$ (tube exit where $T_m = T_e$) gives

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p} \qquad \textbf{(19–55)}$$

where $A_s = pL$ is the surface area of the tube and h is the constant *average* convection heat transfer coefficient. Taking the exponential of both sides and solving for T_e gives the following relation which is very useful for the determination of the *mean fluid temperature at the tube exit:*

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \qquad \textbf{(19–56)}$$

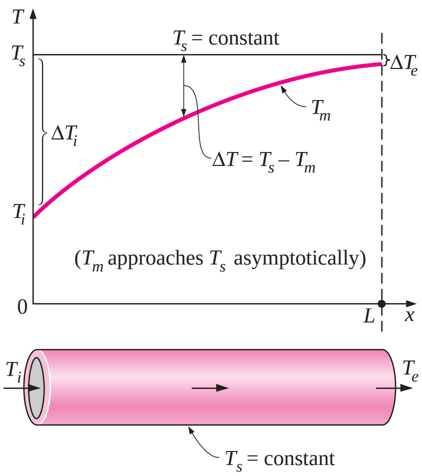
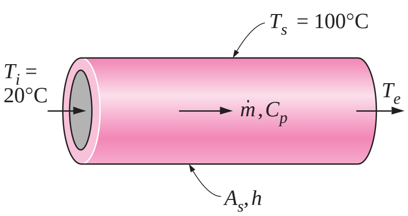


FIGURE 19–27
The variation of the *mean fluid* temperature along the tube for the case of constant temperature.



NTU = $hA_s/\dot{m}C_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

FIGURE 19–28
An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

This relation can also be used to determine the mean fluid temperature $T_m(x)$ at any x by replacing $A_s = pL$ by px.

Note that the temperature difference between the fluid and the surface *decays exponentially* in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_x/\dot{m}C_p$, as shown in Fig. 19–27. This dimensionless parameter is called the *number of transfer units,* denoted by NTU, and is a measure of the effectiveness of the heat transfer systems. For NTU > 5, the exit temperature of the fluid becomes almost equal to the surface temperature, $T_e \approx T_s$ (Fig. 19–28). Noting that the fluid temperature can approach the surface temperature but cannot cross it, an NTU of about 5 indicates that the limit is reached for heat transfer, and the heat transfer will not increase no matter how much we extend the length of the tube. A small value of NTU, on the other hand, indicates more opportunities for heat transfer, and the heat transfer will continue increasing as the tube length is increased. A large NTU and thus a large heat transfer surface area (which means a large tube) may be desirable from a heat transfer point of view, but it may be unacceptable from an economic point of view. The selection of heat transfer equipment usually reflects a compromise between heat transfer performance and cost.

Solving Eq. 19–55 for $\dot{m}C_p$ gives

$$\dot{m}C_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]} \tag{19–57}$$

Substituting this into Eq. 19–41, we obtain

$$\dot{Q} = hA_s\Delta T_{\ln} \tag{19–58}$$

where

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{19–59}$$

is the **logarithmic mean temperature difference.** Note that $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temperature differences between the surface and the fluid at the inlet and the exit of the tube, respectively. This $\Delta T_{\ln}$ relation appears to be prone to misuse, but it is practically fail-safe, since using T_i in place of T_e and vice versa in the numerator and/or the denominator will, at most, affect the sign, not the magnitude. Also, it can be used for both heating ($T_s > T_i$ and T_e) and cooling ($T_s < T_i$ and T_e) of a fluid in a tube.

The logarithmic mean temperature difference $\Delta T_{\ln}$ is obtained by tracing the actual temperature profile of the fluid along the tube, and is an *exact* representation of the *average temperature difference* between the fluid and the surface. It truly reflects the exponential decay of the local temperature difference. When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_e differs from ΔT_i by greater amounts. Therefore, we should always use the logarithmic mean temperature difference when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature T_s.

EXAMPLE 19–6 Heating of Water in a Tube by Steam

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m^2 · C, determine the length of the tube required in order to heat the water to 115°C (Fig. 19–29).

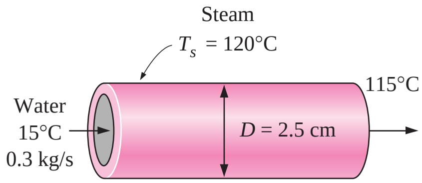


FIGURE 19–29
Schematic for Example 19–6.

SOLUTION Water is heated by steam in a circular tube. The tube length required to heat the water to a specified temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Fluid properties are constant. **3** The convection heat transfer coefficient is constant. **4** The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties The specific heat of water at the bulk mean temperature of (15 + 115)/2 = 65°C is 4187 J/kg · °C. The heat of condensation of steam at 120°C is 2203 kJ/kg (Table A–15).

Analysis Knowing the inlet and exit temperatures of water, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.3 \text{ kg/s})(4.187 \text{ kJ/kg} \cdot °\text{C})(115°\text{C} - 15°\text{C}) = 125.6 \text{ kW}$$

The logarithmic mean temperature difference is

$$\Delta T_e = T_s - T_e = 120°\text{C} - 115°\text{C} = 5°\text{C}$$
$$\Delta T_i = T_s - T_i = 120°\text{C} - 15°\text{C} = 105°\text{C}$$
$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} = \frac{5 - 105}{\ln(5/105)} = 32.85°\text{C}$$

The heat transfer surface area is

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{125.6 \text{ kW}}{(0.8 \text{ kW/m}^2 \cdot °\text{C})(32.85°\text{C})} = 4.78 \text{ m}^2$$

Then the required length of tube becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{4.78 \text{ m}^2}{\pi(0.025 \text{ m})} = \mathbf{61 \text{ m}}$$

Discussion The bulk mean temperature of water during this heating process is 65°C, and thus the *arithmetic* mean temperature difference is ΔT_{am} = 120 − 65 = 55°C. Using ΔT_{am} instead of $\Delta T_{\ln}$ would give L = 36 m, which is grossly in error. This shows the importance of using the logarithmic mean temperature in calculations.

19–7 ▪ LAMINAR FLOW IN TUBES

Reconsider steady laminar flow of a fluid in a circular tube of radius R. The fluid properties ρ, k, and C_p are constant, and the work done by viscous stresses is negligible. The fluid flows along the x-axis with velocity $\mathcal{V}$. The