

23-5 ■ THE EFFECTIVENESS-NTU METHOD

The log mean temperature difference (LMTD) method discussed in Section 23-4 is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. Once ΔT_{lm} , the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from

$$\dot{Q} = UA_s \Delta T_{\text{lm}}$$

Therefore, the LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to *select* a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_{lm} and the correction factor F , if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U .
5. Calculate the heat transfer surface area A_s .

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A_s .

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified. The heat transfer surface area A of the heat exchanger in this case is known, but the *outlet temperatures* are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

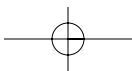
The LMTD method could still be used for this alternative problem, but the procedure would require tedious iterations, and thus it is not practical. In an attempt to eliminate the iterations from the solution of such problems, Kays and London came up with a method in 1955 called the **effectiveness-NTU method**, which greatly simplified heat exchanger analysis.

This method is based on a dimensionless parameter called the **heat transfer effectiveness** ε , defined as

$$\varepsilon = \frac{\dot{Q}}{Q_{\text{max}}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \quad (23-29)$$

The *actual* heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c(T_{c, \text{out}} - T_{c, \text{in}}) = C_h(T_{h, \text{in}} - T_{h, \text{out}}) \quad (23-30)$$





where $C_c = \dot{m}_c C_{pc}$ and $C_h = \dot{m}_h C_{ph}$ are the heat capacity rates of the cold and the hot fluids, respectively.

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is,

$$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} \quad (23-31)$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the *smaller* heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (Fig. 23–23)

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \quad (23-32)$$

where C_{\min} is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$. This is further clarified by Example 23–7.

EXAMPLE 23–7 Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s , where it is heated by a hot-water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s . Assuming the specific heat of water to remain constant at $C_p = 4.18 \text{ kJ/kg} \cdot ^\circ\text{C}$, determine the maximum heat transfer rate and the outlet temperatures of the cold- and the hot-water streams for this limiting case.

SOLUTION Cold- and hot-water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

Assumptions 1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. 3 Changes in the kinetic and potential energies of fluid streams are negligible. 4 Heat transfer coefficients and fouling factors are constant and uniform. 5 The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

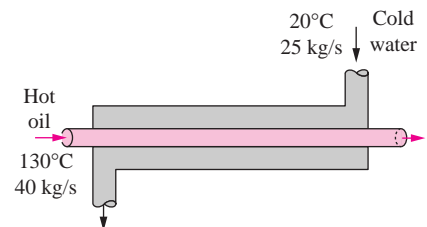
Properties The specific heat of water is given to be $C_p = 4.18 \text{ kJ/kg} \cdot ^\circ\text{C}$.

Analysis A schematic of the heat exchanger is given in Fig. 23–24. The heat capacity rates of the hot and cold fluids are determined from

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C}) = 8.36 \text{ kW}/^\circ\text{C}$$

and

$$C_c = \dot{m}_c C_{pc} = (8 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C}) = 33.4 \text{ kW}/^\circ\text{C}$$



$$C_c = \dot{m}_c C_{pc} = 104.5 \text{ kW}/^\circ\text{C}$$

$$C_h = \dot{m}_h C_{ph} = 92 \text{ kW}/^\circ\text{C}$$

$$C_{\min} = 92 \text{ kW}/^\circ\text{C}$$

$$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = 110^\circ\text{C}$$

$$\dot{Q}_{\max} = C_{\min} \Delta T_{\max} = 10,120 \text{ kW}$$

FIGURE 23–23

The determination of the maximum rate of heat transfer in a heat exchanger.

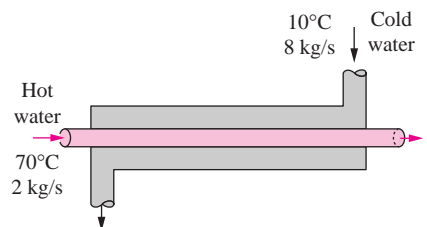
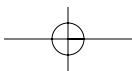


FIGURE 23–24

Schematic for Example 23–7.



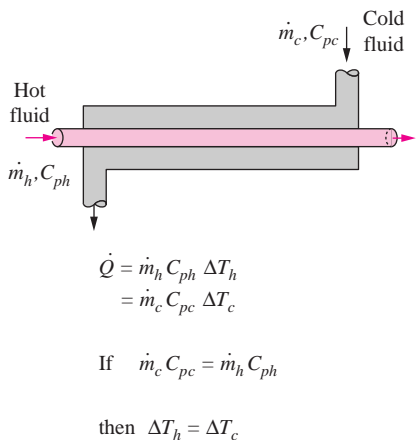


FIGURE 23–25

The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the mass flow rates and the specific heats of the hot and cold fluids are identical.

Therefore

$$C_{\min} = C_h = 8.36 \text{ kW/}^\circ\text{C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Eq. 23–32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \\ &= (8.36 \text{ kW/}^\circ\text{C})(70 - 10)^\circ\text{C} \\ &= \mathbf{502 \text{ kW}}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area.

The maximum temperature difference in this heat exchanger is $\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} = (70 - 10)^\circ\text{C} = 60^\circ\text{C}$. Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\begin{aligned}\dot{Q} &= C_c(T_{c,\text{out}} - T_{c,\text{in}}) \longrightarrow T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c} = 10^\circ\text{C} + \frac{502 \text{ kW}}{33.4 \text{ kW/}^\circ\text{C}} = \mathbf{25^\circ\text{C}} \\ \dot{Q} &= C_h(T_{h,\text{in}} - T_{h,\text{out}}) \longrightarrow T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} = 70^\circ\text{C} - \frac{502 \text{ kW}}{8.38 \text{ kW/}^\circ\text{C}} = \mathbf{10^\circ\text{C}}\end{aligned}$$

Discussion Note that the hot water is cooled to the limit of 10°C (the inlet temperature of the cold-water stream), but the cold water is heated to 25°C only when maximum heat transfer occurs in the heat exchanger. This is not surprising, since the mass flow rate of the hot water is only one-fourth that of the cold water, and, as a result, the temperature of the cold water increases by 0.25°C for each 1°C drop in the temperature of the hot water.

You may be tempted to think that the cold water should be heated to 70°C in the limiting case of maximum heat transfer. But this will require the temperature of the hot water to drop to -170°C (below 10°C), which is impossible. Therefore, heat transfer in a heat exchanger reaches its maximum value when the fluid with the smaller heat capacity rate (or the smaller mass flow rate when both fluids have the same specific heat value) experiences the maximum temperature change. This example explains why we use C_{\min} in the evaluation of \dot{Q}_{\max} instead of C_{\max} .

We can show that the hot water will leave at the inlet temperature of the cold water and vice versa in the limiting case of maximum heat transfer when the mass flow rates of the hot- and cold-water streams are identical (Fig. 23–25). We can also show that the outlet temperature of the cold water will reach the 70°C limit when the mass flow rate of the hot water is greater than that of the cold water.

The determination of \dot{Q}_{\max} requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates*, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate \dot{Q} can be determined from

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \quad (23-33)$$

Therefore, the effectiveness of a heat exchanger enables us to determine the heat transfer rate without knowing the *outlet temperatures* of the fluids.

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement*. Therefore, different types of heat exchangers have different effectiveness relations. Below we illustrate the development of the effectiveness ε relation for the double-pipe *parallel-flow* heat exchanger.

Equation 23–23 developed in Section 23–4 for a parallel-flow heat exchanger can be rearranged as

$$\ln \frac{T_{h, \text{out}} - T_{c, \text{out}}}{T_{h, \text{in}} - T_{c, \text{in}}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \quad (23-34)$$

Also, solving Eq. 23–30 for $T_{h, \text{out}}$ gives

$$T_{h, \text{out}} = T_{h, \text{in}} - \frac{C_c}{C_h} (T_{c, \text{out}} - T_{c, \text{in}}) \quad (23-35)$$

Substituting this relation into Eq. 23–34 after adding and subtracting $T_{c, \text{in}}$ gives

$$\ln \frac{T_{h, \text{in}} - T_{c, \text{in}} + T_{c, \text{in}} - T_{c, \text{out}} - \frac{C_c}{C_h} (T_{c, \text{out}} - T_{c, \text{in}})}{T_{h, \text{in}} - T_{c, \text{in}}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)$$

which simplifies to

$$\ln \left[1 - \left(1 + \frac{C_c}{C_h}\right) \frac{T_{c, \text{out}} - T_{c, \text{in}}}{T_{h, \text{in}} - T_{c, \text{in}}} \right] = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \quad (23-36)$$

We now manipulate the definition of effectiveness to obtain

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{C_c(T_{c, \text{out}} - T_{c, \text{in}})}{C_{\min}(T_{h, \text{in}} - T_{c, \text{in}})} \longrightarrow \frac{T_{c, \text{out}} - T_{c, \text{in}}}{T_{h, \text{in}} - T_{c, \text{in}}} = \varepsilon \frac{C_{\min}}{C_c}$$

Substituting this result into Eq. 23–36 and solving for ε gives the following relation for the effectiveness of a *parallel-flow* heat exchanger:

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp \left[-\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \right]}{\left(1 + \frac{C_c}{C_h}\right) \frac{C_{\min}}{C_c}} \quad (23-37)$$

Taking either C_c or C_h to be C_{\min} (both approaches give the same result), the relation above can be expressed more conveniently as

$$\varepsilon_{\text{parallel flow}} = \frac{1 - \exp \left[-\frac{UA_s}{C_{\min}} \left(1 + \frac{C_{\min}}{C_{\max}}\right) \right]}{1 + \frac{C_{\min}}{C_{\max}}} \quad (23-38)$$

Again C_{\min} is the *smaller* heat capacity ratio and C_{\max} is the larger one, and it makes no difference whether C_{\min} belongs to the hot or cold fluid.

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group UA_s/C_{\min} . This quantity is called the **number of transfer units NTU** and is expressed as

$$NTU = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}C_p)_{\min}} \quad (23-39)$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s . Therefore, for specified values of U and C_{\min} , the value of NTU is a *measure of the heat transfer surface area* A_s . Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the **capacity ratio c** as

$$c = \frac{C_{\min}}{C_{\max}} \quad (23-40)$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c . That is,

$$\varepsilon = \text{function}(UA_s/C_{\min}, C_{\min}/C_{\max}) = \text{function}(NTU, c)$$

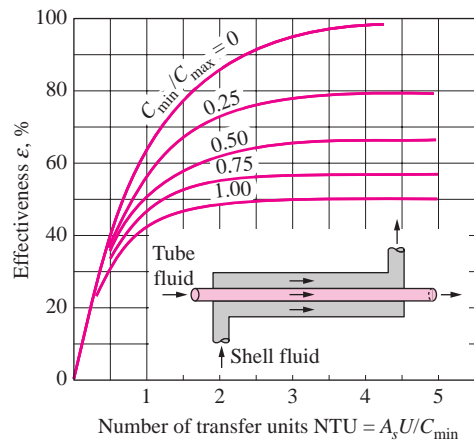
Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table 23-4. The effectivenesses of some common types of heat exchangers are also plotted in Fig. 23-26. More

TABLE 23-4

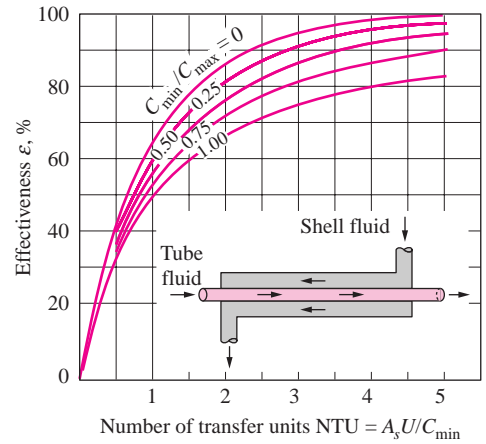
Effectiveness relations for heat exchangers: $NTU = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London)

Heat exchanger type	Effectiveness relation
1 <i>Double pipe:</i>	
Parallel-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 - c)]}{1 - c \exp[-NTU(1 - c)]}$
2 <i>Shell-and-tube:</i>	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2 \left\{ 1 + c + \sqrt{1 + c^2} \frac{1 + \exp[-NTU\sqrt{1 + c^2}]}{1 - \exp[-NTU\sqrt{1 + c^2}]} \right\}^{-1}$
3 <i>Cross-flow (single-pass)</i>	
Both fluids unmixed	$\varepsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} [\exp(-c NTU^{0.78}) - 1] \right\}$
C_{\max} mixed, C_{\min} unmixed	$\varepsilon = \frac{1}{c} (1 - \exp\{1 - c[1 - \exp(-NTU)]\})$
C_{\min} mixed, C_{\max} unmixed	$\varepsilon = 1 - \exp \left\{ -\frac{1}{c} [1 - \exp(-c NTU)] \right\}$
4 <i>All heat exchangers with $c = 0$</i>	$\varepsilon = 1 - \exp(-NTU)$

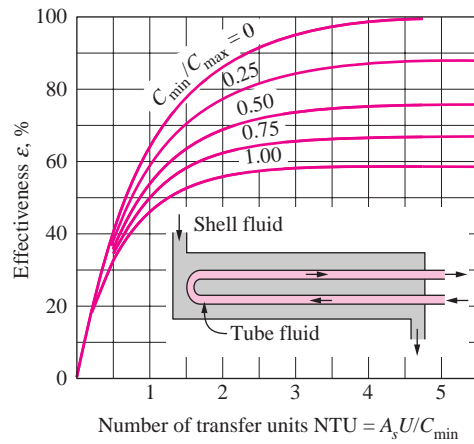
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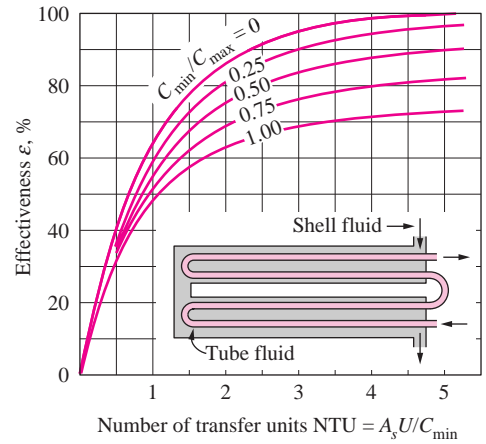
(a) Parallel-flow



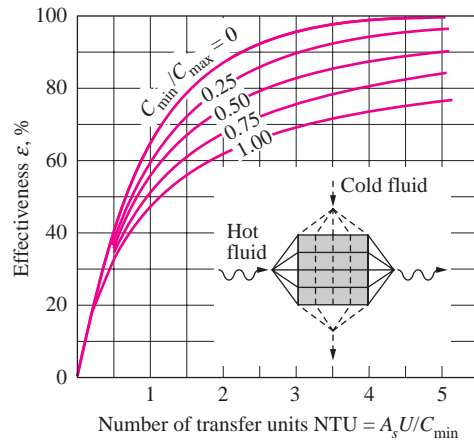
(b) Counter-flow



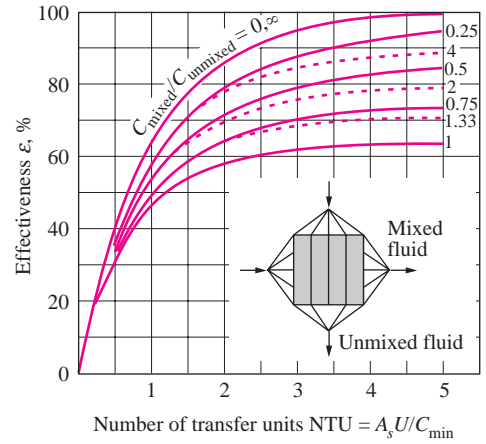
(c) One-shell pass and 2, 4, 6, ... tube passes



(d) Two-shell passes and 4, 8, 12, ... tube passes



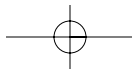
(e) Cross-flow with both fluids unmixed



(f) Cross-flow with one fluid mixed and the other unmixed

FIGURE 23-26

Effectiveness for heat exchangers (from Kays and London).



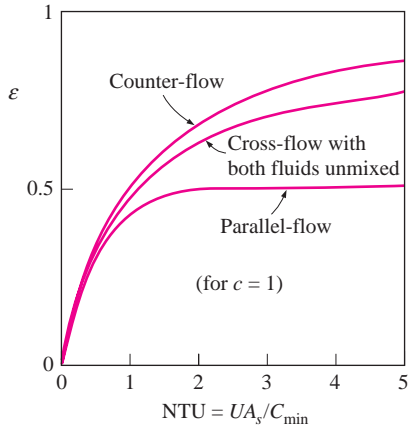


FIGURE 23-27

For a specified NTU and capacity ratio c , the counter-flow heat exchanger has the highest effectiveness and the parallel-flow the lowest.

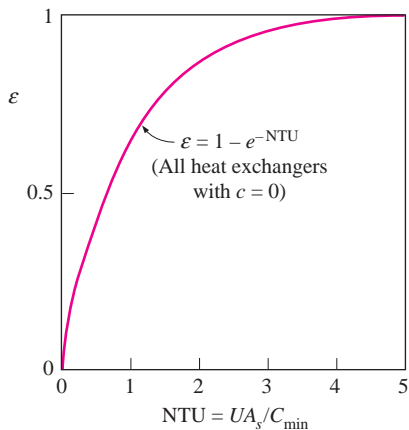


FIGURE 23-28

The effectiveness relation reduces to $\varepsilon = \varepsilon_{\max} = 1 - \exp(-NTU)$ for all heat exchangers when the capacity ratio $c = 0$.

extensive effectiveness charts and relations are available in the literature. The dashed lines in Fig. 23–26*f* are for the case of C_{\min} unmixed and C_{\max} mixed and the solid lines are for the opposite case. The analytic relations for the effectiveness give more accurate results than the charts, since reading errors in charts are unavoidable, and the relations are very suitable for computerized analysis of heat exchangers.

We make these observations from the effectiveness relations and charts already given:

1. The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about $NTU = 1.5$) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness. Thus, a heat exchanger with a very high effectiveness may be highly desirable from a heat transfer point of view but rather undesirable from an economical point of view.
2. For a given NTU and capacity ratio $c = C_{\min}/C_{\max}$, the counter-flow heat exchanger has the *highest* effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. As you might expect, the lowest effectiveness values are encountered in parallel-flow heat exchangers (Fig. 23–27).
3. The effectiveness of a heat exchanger is independent of the capacity ratio c for NTU values of less than about 0.3.
4. The value of the capacity ratio c ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for $c = 0$ and a *minimum* for $c = 1$. The case $c = C_{\min}/C_{\max} \rightarrow 0$ corresponds to $C_{\max} \rightarrow \infty$, which is realized during a phase-change process in a *condenser* or *boiler*. All effectiveness relations in this case reduce to

$$\varepsilon = \varepsilon_{\max} = 1 - \exp(-NTU) \quad (23-41)$$

regardless of the type of heat exchanger (Fig. 23–28). Note that the temperature of the condensing or boiling fluid remains constant in this case. The effectiveness is the *lowest* in the other limiting case of $c = C_{\min}/C_{\max} = 1$, which is realized when the heat capacity rates of the two fluids are equal.

Once the quantities $c = C_{\min}/C_{\max}$ and $NTU = UA_s/C_{\min}$ have been evaluated, the effectiveness ε can be determined from either the charts or (preferably) the effectiveness relation for the specified type of heat exchanger. Then the rate of heat transfer \dot{Q} and the outlet temperatures $T_{h, \text{out}}$ and $T_{c, \text{out}}$ can be determined from Eqs. 23–33 and 23–30, respectively. Note that the analysis of heat exchangers with unknown outlet temperatures is a straightforward matter with the effectiveness–NTU method but requires rather tedious iterations with the LMTD method.

We mentioned earlier that when all the inlet and outlet temperatures are specified, the *size* of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can also be determined from the effectiveness–NTU method by first evaluating the effectiveness ε from its definition (Eq. 23–29) and then the NTU from the appropriate NTU relation in Table 23–5.

TABLE 23-5

NTU relations for heat exchangers: $NTU = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London)

Heat exchanger type	NTU relation
1 <i>Double-pipe:</i> Parallel-flow	$NTU = -\frac{\ln[1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$NTU = \frac{1}{c - 1} \ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right)$
2 <i>Shell and tube:</i> One-shell pass 2, 4, . . . tube passes	$NTU = -\frac{1}{\sqrt{1 + c^2}} \ln\left(\frac{2/\varepsilon - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon - 1 - c + \sqrt{1 + c^2}}\right)$
3 <i>Cross-flow (single-pass):</i> C_{\max} mixed, C_{\min} unmixed	$NTU = -\ln\left[1 + \frac{\ln(1 - \varepsilon c)}{c}\right]$
C_{\min} mixed, C_{\max} unmixed	$NTU = -\frac{\ln[c \ln(1 - \varepsilon) + 1]}{c}$
4 <i>All heat exchangers</i> with $c = 0$	$NTU = -\ln(1 - \varepsilon)$

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Note that the relations in Table 23-5 are equivalent to those in Table 23-4. Both sets of relations are given for convenience. The relations in Table 23-4 give the effectiveness directly when NTU is known, and the relations in Table 23-5 give the NTU directly when the effectiveness ε is known.

EXAMPLE 23-8 Using the Effectiveness–NTU Method

Repeat Example 23-4, which was solved with the LMTD method, using the effectiveness–NTU method.

SOLUTION The schematic of the heat exchanger is redrawn in Fig. 23-29, and the same assumptions are utilized.

Analysis In the effectiveness–NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot ^\circ\text{C}) = 8.62 \text{ kW}/^\circ\text{C}$$

$$C_c = \dot{m}_c C_{pc} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C}) = 5.02 \text{ kW}/^\circ\text{C}$$

Therefore,

$$C_{\min} = C_c = 5.02 \text{ kW}/^\circ\text{C}$$

and

$$c = C_{\min}/C_{\max} = 5.02/8.62 = 0.583$$

Then the maximum heat transfer rate is determined from Eq. 23-32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h, \text{in}} - T_{c, \text{in}}) \\ &= (5.02 \text{ kW}/^\circ\text{C})(160 - 20)^\circ\text{C} \\ &= 702.8 \text{ kW}\end{aligned}$$

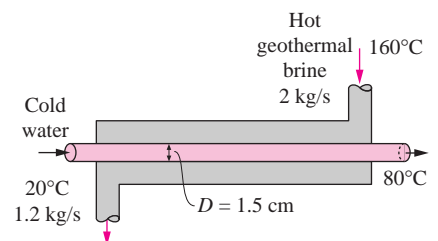


FIGURE 23-29
Schematic for Example 23-8.

That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C})(80 - 20)^\circ\text{C} = 301.0 \text{ kW}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{Q_{\text{max}}} = \frac{301.0 \text{ kW}}{702.8 \text{ kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counter-flow heat exchanger can be determined from Fig. 23–26*b* or the appropriate relation from Table 23–5. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{c - 1} \ln \left(\frac{\varepsilon - 1}{\varepsilon c - 1} \right) = \frac{1}{0.583 - 1} \ln \left(\frac{0.428 - 1}{0.428 \times 0.583 - 1} \right) = 0.651$$

Then the heat transfer surface area becomes

$$\text{NTU} = \frac{UA_s}{C_{\text{min}}} \longrightarrow A_s = \frac{\text{NTU} C_{\text{min}}}{U} = \frac{(0.651)(5020 \text{ W}/^\circ\text{C})}{640 \text{ W}/\text{m}^2 \cdot ^\circ\text{C}} = 5.11 \text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = \mathbf{108 \text{ m}}$$

Discussion Note that we obtained the same result with the effectiveness–NTU method in a systematic and straightforward manner.

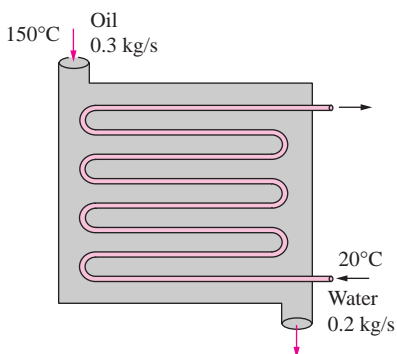


FIGURE 23–30
Schematic for Example 23–9.

EXAMPLE 23–9 Cooling Hot Oil by Water in a Multipass Heat Exchanger

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is $310 \text{ W}/\text{m}^2 \cdot ^\circ\text{C}$. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C , respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

SOLUTION Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. 3 The thickness of the tube is negligible since it is thin-walled. 4 Changes in the kinetic and potential energies of fluid streams are negligible. 5 The overall heat transfer coefficient is constant and uniform.

Analysis The schematic of the heat exchanger is given in Fig. 23–30. The outlet temperatures are not specified, and they cannot be determined from an

energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε -NTU method is indicated. The first step in the ε -NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot ^\circ\text{C}) = 0.639 \text{ kW}/^\circ\text{C}$$

$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C}) = 0.836 \text{ kW}/^\circ\text{C}$$

Therefore,

$$C_{\min} = C_h = 0.639 \text{ kW}/^\circ\text{C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is determined from Eq. 23-32 to be

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \\ &= (0.639 \text{ kW}/^\circ\text{C})(150 - 20)^\circ\text{C} = 83.1 \text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is

$$A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2 \cdot ^\circ\text{C})(1.76 \text{ m}^2)}{639 \text{ W}/^\circ\text{C}} = 0.853$$

The effectiveness of this heat exchanger corresponding to $c = 0.764$ and $\text{NTU} = 0.853$ is determined from Fig. 23-26c to be

$$\varepsilon = 0.47$$

We could also determine the effectiveness from the third relation in Table 23-4 more accurately but with more labor. Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = \mathbf{39.1 \text{ kW}}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\begin{aligned}\dot{Q} &= C_c(T_{c,\text{out}} - T_{c,\text{in}}) \longrightarrow T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c} \\ &= 20^\circ\text{C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW}/^\circ\text{C}} = \mathbf{66.8^\circ\text{C}}\end{aligned}$$

$$\begin{aligned}\dot{Q} &= C_h(T_{h,\text{in}} - T_{h,\text{out}}) \longrightarrow T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} \\ &= 150^\circ\text{C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW}/^\circ\text{C}} = \mathbf{88.8^\circ\text{C}}\end{aligned}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.