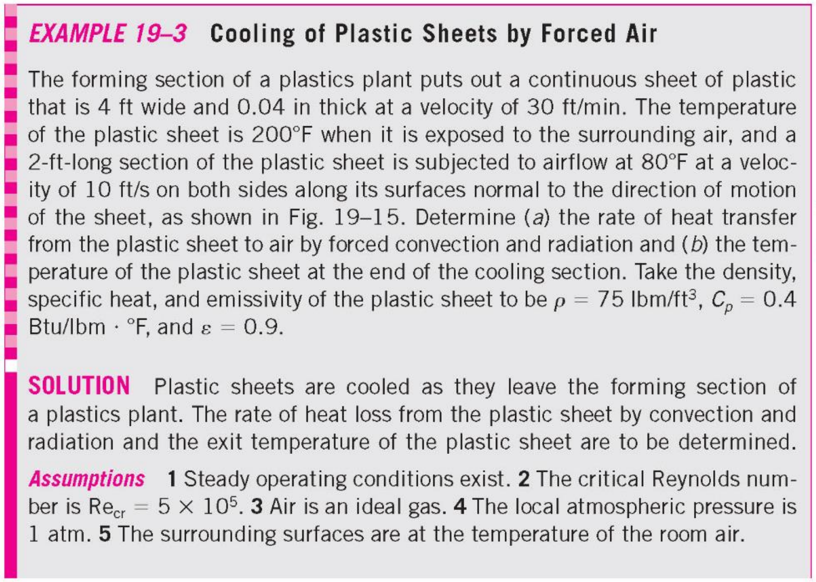
Mech 262 – Forced Convection Example (Your Text: Example 19-2, Page 784)

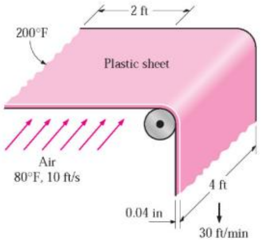


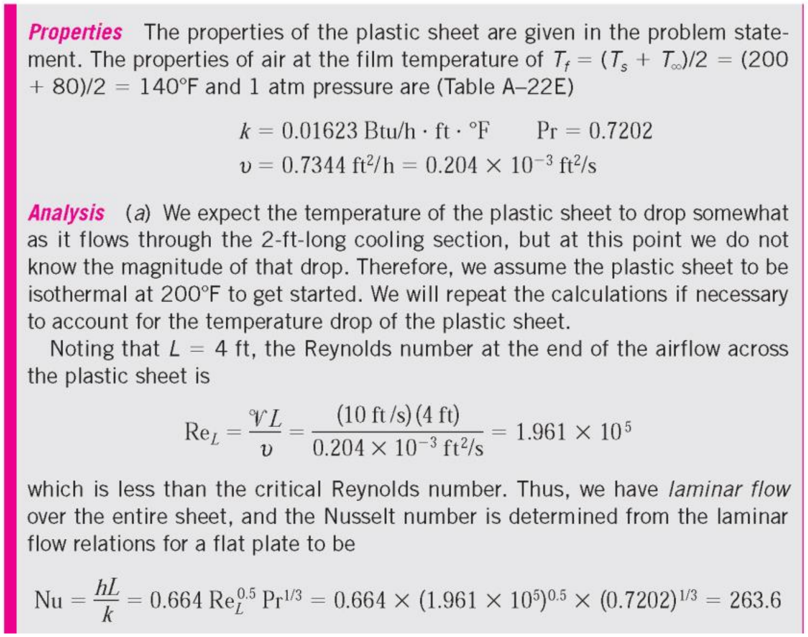
EXAMPLE 19–3 Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 4 ft wide and 0.04 in thick at a velocity of 30 ft/min. The temperature of the plastic sheet is 200°F when it is exposed to the surrounding air, and a 2-ft-long section of the plastic sheet is subjected to airflow at 80°F at a velocity of 10 ft/s on both sides along its surfaces normal to the direction of motion of the sheet, as shown in Fig. 19–15. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\rho = 75$ lbm/ft³, $C_p = 0.4$ Btu/lbm · °F, and $\varepsilon = 0.9$.

SOLUTION Plastic sheets are cooled as they leave the forming section of a plastics plant. The rate of heat loss from the plastic sheet by convection and radiation and the exit temperature of the plastic sheet are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$. **3** Air is an ideal gas. **4** The local atmospheric pressure is 1 atm. **5** The surrounding surfaces are at the temperature of the room air.





Properties The properties of the plastic sheet are given in the problem statement. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (200 + 80)/2 = 140°\mathrm{F}$ and 1 atm pressure are (Table A–22E)

$$k = 0.01623\ \mathrm{Btu/h \cdot ft \cdot °F} \qquad \mathrm{Pr} = 0.7202$$
$$\upsilon = 0.7344\ \mathrm{ft^2/h} = 0.204 \times 10^{-3}\ \mathrm{ft^2/s}$$

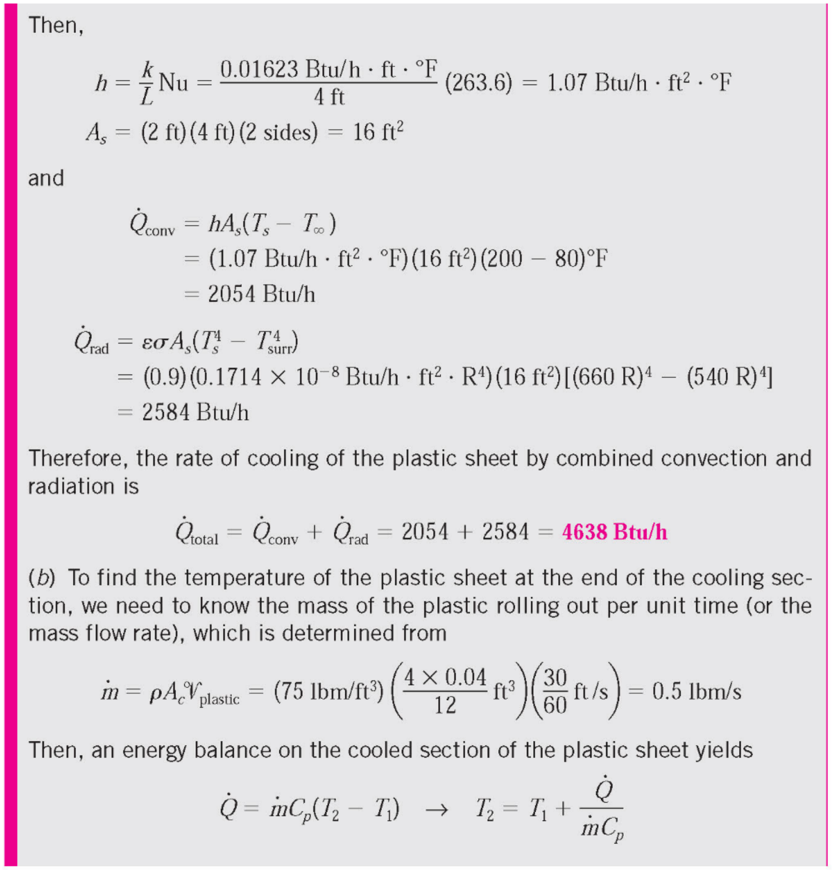
Analysis (*a*) We expect the temperature of the plastic sheet to drop somewhat as it flows through the 2-ft-long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at 200°F to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet.

Noting that $L = 4$ ft, the Reynolds number at the end of the airflow across the plastic sheet is

$$\mathrm{Re}_L = \frac{\mathcal{V}L}{\upsilon} = \frac{(10\ \mathrm{ft/s})(4\ \mathrm{ft})}{0.204 \times 10^{-3}\ \mathrm{ft^2/s}} = 1.961 \times 10^5$$

which is less than the critical Reynolds number. Thus, we have *laminar flow* over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} = 0.664 \times (1.961 \times 10^5)^{0.5} \times (0.7202)^{1/3} = 263.6$$



Then,

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.01623\ \mathrm{Btu/h \cdot ft \cdot °F}}{4\ \mathrm{ft}}(263.6) = 1.07\ \mathrm{Btu/h \cdot ft^2 \cdot °F}$$
$$A_s = (2\ \mathrm{ft})(4\ \mathrm{ft})(2\ \mathrm{sides}) = 16\ \mathrm{ft^2}$$

and

$$\begin{aligned}\dot{Q}_{\mathrm{conv}} &= hA_s(T_s - T_\infty)\\ &= (1.07\ \mathrm{Btu/h \cdot ft^2 \cdot °F})(16\ \mathrm{ft^2})(200 - 80)°\mathrm{F}\\ &= 2054\ \mathrm{Btu/h}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\mathrm{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\mathrm{surr}}^4)\\ &= (0.9)(0.1714 \times 10^{-8}\ \mathrm{Btu/h \cdot ft^2 \cdot R^4})(16\ \mathrm{ft^2})[(660\ \mathrm{R})^4 - (540\ \mathrm{R})^4]\\ &= 2584\ \mathrm{Btu/h}\end{aligned}$$

Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is

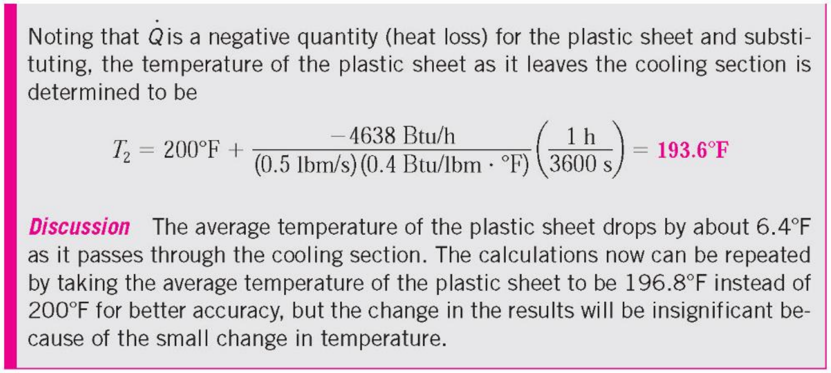
$$\dot{Q}_{\mathrm{total}} = \dot{Q}_{\mathrm{conv}} + \dot{Q}_{\mathrm{rad}} = 2054 + 2584 = \mathbf{4638\ Btu/h}$$

(*b*) To find the temperature of the plastic sheet at the end of the cooling section, we need to know the mass of the plastic rolling out per unit time (or the mass flow rate), which is determined from

$$\dot{m} = \rho A_c \mathcal{V}_{\mathrm{plastic}} = (75\ \mathrm{lbm/ft^3})\left(\frac{4 \times 0.04}{12}\ \mathrm{ft^3}\right)\left(\frac{30}{60}\ \mathrm{ft/s}\right) = 0.5\ \mathrm{lbm/s}$$

Then, an energy balance on the cooled section of the plastic sheet yields

$$\dot{Q} = \dot{m}C_p(T_2 - T_1) \quad \rightarrow \quad T_2 = T_1 + \frac{\dot{Q}}{\dot{m}C_p}$$



Noting that $\dot{Q}$ is a negative quantity (heat loss) for the plastic sheet and substituting, the temperature of the plastic sheet as it leaves the cooling section is determined to be

$$T_2 = 200°\mathrm{F} + \frac{-4638\ \mathrm{Btu/h}}{(0.5\ \mathrm{lbm/s})(0.4\ \mathrm{Btu/lbm \cdot °F})}\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = \mathbf{193.6°F}$$

Discussion The average temperature of the plastic sheet drops by about 6.4°F as it passes through the cooling section. The calculations now can be repeated by taking the average temperature of the plastic sheet to be 196.8°F instead of 200°F for better accuracy, but the change in the results will be insignificant because of the small change in temperature.