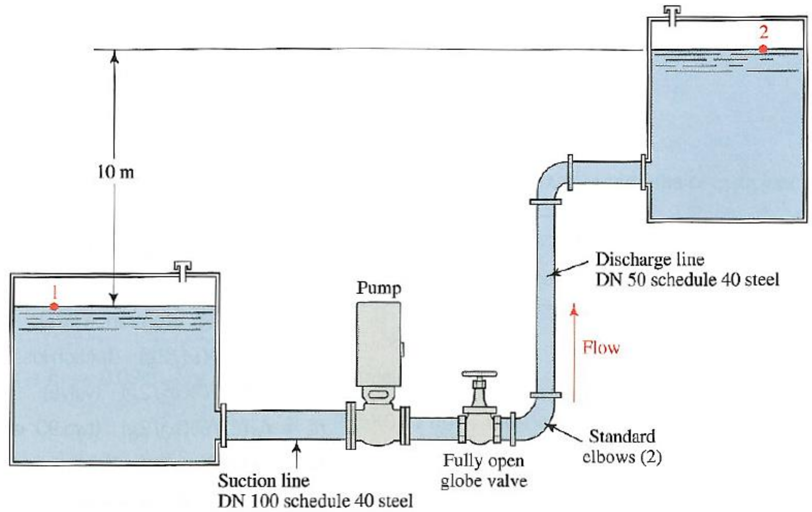
Meng 263 – Fluids & Heat Transfer

Class I – Series Pipe Flow – Example 1



PROGRAMMED EXAMPLE PROBLEM

Example Problem 11.1

Calculate the power supplied to the pump shown in Fig. 11.2 if its efficiency is 76 percent. Methyl alcohol at 25°C is flowing at the rate of 54.0 m^3/h. The suction line is a standard DN 100 Schedule 40 steel pipe, 15 m long. The total length of DN 50 Schedule 40 steel pipe in the discharge line is 200 m. Assume that the entrance from reservoir 1 is through a square-edged inlet and that the elbows are standard. The valve is a fully open globe valve.

To begin the solution, write the energy equation for the system.

Using the surfaces of the reservoirs as the reference points, you should have

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

Because $p_1 = p_2 = 0$ and v_1 and v_2 are approximately zero, the equation can be simplified to

$$z_1 + h_A - h_L = z_2$$

Because the objective of the problem is to calculate the power supplied to the pump, solve now for the total head on the pump, h_A.

The total head is

$$h_A = z_2 - z_1 + h_L$$

There are six components to the total energy loss. List them and write the formula for evaluating each one.

Your list should include the following items. The subscript s indicates the suction line and the subscript d indicates the discharge line:

$h_1 = K(v_s^2/2g)$ (entrance loss)
$h_2 = f_s(L/D)(v_s^2/2g)$ (friction loss in suction line)
$h_3 = f_{dT}(L_e/D)(v_d^2/2g)$ (valve)
$h_4 = f_{dT}(L_e/D)(v_d^2/2g)$ (two 90° elbows)
$h_5 = f_d(L/D)(v_d^2/2g)$ (friction loss in discharge line)
$h_6 = 1.0(v_d^2/2g)$ (exit loss)

Because the velocity head in the suction or discharge line is required for each energy loss, calculate these values now.

You should have $v_s^2/2g = 0.17$ m and $v_d^2/2g = 2.44$ m, found as follows:

$$Q = \frac{54.0\ \text{m}^3}{\text{hr}} \times \frac{1\ \text{hr}}{3600\ \text{s}} = 0.015\ \text{m}^3/\text{s}$$

$$v_s = \frac{Q}{A_s} = \frac{0.015\ \text{m}^3}{\text{s}} \times \frac{1}{8.213 \times 10^{-3}\ \text{m}^2} = 1.83\ \text{m/s}$$

$$\frac{v_s^2}{2g} = \frac{(1.83)^2}{2(9.81)}\text{m} = 0.17\ \text{m}$$

$$v_d = \frac{Q}{A_d} = \frac{0.015\ \text{m}^3}{\text{s}} \times \frac{1}{2.168 \times 10^{-3}\ \text{m}^2} = 6.92\ \text{m/s}$$

$$\frac{v_d^2}{2g} = \frac{(6.92)^2}{2(9.81)}\text{m} = 2.44\ \text{m}$$

To determine the friction losses in the suction line and the discharge line and the minor losses in the discharge line, we need the Reynolds number, relative roughness, and friction factor for each pipe, and the friction factor in the zone of complete turbulence for the discharge line that contains a valve and pipe fittings. Find these values now.

For methyl alcohol at 25°C, $\rho = 789\ \text{kg/m}^3$ and $\eta = 5.60 \times 10^{-4}$ Pa·s. Then, in the suction line, we have

$$N_R = \frac{vD\rho}{\eta} = \frac{(1.83)(0.1023)(789)}{5.60 \times 10^{-4}} = 2.64 \times 10^5$$

Because the flow is turbulent, the value of f_s must be evaluated from the Moody diagram, Fig. 8.7. For steel pipe, $\varepsilon = 4.6 \times 10^{-5}$ m. Write

$$D/\varepsilon = 0.1023/(4.6 \times 10^{-5}) = 2224$$
$$N_R = 2.64 \times 10^5$$

Then $f_s = 0.018$.

In the discharge line, we have

$$N_R = \frac{vD\rho}{\eta} = \frac{(6.92)(0.0525)(789)}{5.60 \times 10^{-4}} = 5.12 \times 10^5$$

This flow is also turbulent. Evaluating the friction factor f_d gives

$$D/\varepsilon = 0.0525/(4.6 \times 10^{-5}) = 1141$$
$$N_R = 5.12 \times 10^5$$
$$f_d = 0.020$$

We can find from Table 10.5 that $f_{dT} = 0.019$ for the DN 50 discharge pipe in the zone of complete turbulence.

Returning now to the energy loss calculations, evaluate h_1, the entrance loss, in N·m/N or m.

The result is $h_1 = 0.09$ m. For a square-edged inlet, $K = 0.5$ and

$$h_1 = 0.5(v_s^2/2g) = (0.5)(0.17\ \text{m}) = 0.09\ \text{m}$$

Now calculate h_2, the friction loss in the suction line.

The result is $h_2 = 0.45$ m.

$$h_2 = f_s \times \frac{L}{D} \times \frac{v_s^2}{2g} = f_s\left(\frac{15}{0.1023}\right)(0.17)\text{ m}$$

$$h_2 = (0.018)\left(\frac{15}{0.1023}\right)(0.17)\text{ m} = 0.45\text{ m}$$

Now calculate h_3, the energy loss in the valve in the discharge line.

From the data in Chapter 10, the equivalent-length ratio L_e/D for a fully open globe valve is 340. The friction factor is $f_{dT} = 0.019$. Then, we have

$$h_3 = f_{dT} \times \frac{L_e}{D} \times \frac{v_d^2}{2g} = (0.019)(340)(2.44)\text{ m} = 15.76\text{ m}$$

Now calculate h_4, the energy loss in the two 90° elbows.

For standard 90° elbows, $L_e/D = 30$. The value of f_{dT} is 0.019, the same as that used in the preceding panel. Then, we have

$$h_4 = 2f_{dT} \times \frac{L_e}{D} \times \frac{v_d^2}{2g} = (2)(0.019)(30)(2.44)\text{ m} = 2.78\text{ m}$$

Now calculate h_5, the friction loss in the discharge line.

The discharge-line friction loss is

$$h_5 = f_d \times \frac{L}{D} \times \frac{v_d^2}{2g} = (0.020)\left(\frac{200}{0.0525}\right)(2.44)\text{ m} = 185.9\text{ m}$$

Now calculate h_6, the exit loss.

The exit loss is

$$h_6 = 1.0(v_d^2/2g) = 2.44\text{ m}$$

This concludes the calculation of the individual energy losses. The total loss h_L can now be determined.

$$h_L = h_1 + h_2 + h_3 + h_4 + h_5 + h_6$$
$$h_L = (0.09 + 0.45 + 15.76 + 2.78 + 185.9 + 2.44)\text{ m}$$
$$h_L = 207.4\text{ m}$$

From the energy equation the expression for the total head on the pump is

$$h_A = z_2 - z_1 + h_L$$

Then, we have

$$h_A = 10\text{ m} + 207.4\text{ m} = 217.4\text{ m}$$

Now calculate the power supplied to the pump, P_A.

$$\text{Power} = \frac{h_A \gamma Q}{e_M} = \frac{(217.4\text{ m})(7.74 \times 10^3\text{ N/m}^3)(0.015\text{ m}^3\text{/s})}{0.76}$$
$$P_A = 33.2 \times 10^3\text{ N·m/s} = 33.2\text{ kW}$$

This concludes the programmed example problem.