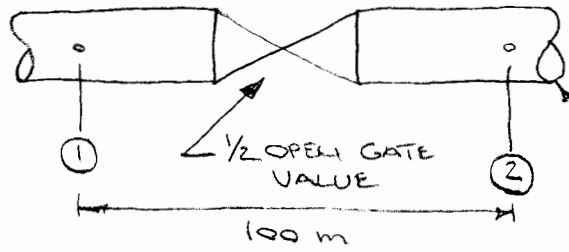
MECH 277 - FLUID MECHANICS II

CLASS II SYSTEM → FIND $\dot{Q}$



ΔP_{1-2} = 50 kPa. WATER AT 5°C

① FIND AN EQ^{n} RELATING V AND f

$$\frac{(P_1 - P_2)}{\gamma} + \cancel{(z_1 - z_2)}^{\,0} + \cancel{\frac{(V_1^2 - V_2^2)}{2g}}^{\,0} - \cancel{h_A}^{\,0} - \cancel{h_R}^{\,0} - h_L = 0$$

$$\therefore \quad \frac{P_1 - P_2}{\gamma} - h_L = 0 \qquad \text{or} \quad h_L =$$

$$\text{or} \quad \underline{h_L = \frac{P_1 - P_2}{\gamma} = \frac{50 \times 10^3}{9.81 \times 10^3} = 5.1 \text{ m}}$$

NOTE: h_L OBTAINED FROM EXTERNAL CONDITIONS.

ALSO RECALL: $h_L = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$ AND $h_L = f\left(\frac{L_E}{D}\right)\left(\frac{V^2}{2g}\right)$ (PIPE; FITTINGS)

$$\text{so} \quad \boxed{h_L = f\left[\Sigma\left(\frac{L}{D}\right) + \Sigma\left(\frac{L_E}{D}\right)\right]\left(V^2/2g\right)}$$

$\Sigma\left(\frac{L_E}{D}\right)$: SUM OF ALL $\left(\frac{L_E}{D}\right)$'S FOR ALL FITTINGS.

$\Sigma\left(\frac{L}{D}\right)$: SUM OF ALL $\left(\frac{L}{D}\right)$'S FOR ALL PIPES.

PROPERTIES

WATER AT 5°C
γ = 9.81 kN/m^3
ρ = 1000 kg/m^3
$\mu = 1.52 \times 10^{-3}$ Pa·s

PIPE: STEEL
$\epsilon = 4.6 \times 10^{-5}$ m
(TABLE 9.1, Pg. 240)
ID = 150 mm

FITTING:
½ OPEN GATE VALVE
$\frac{L_E}{D}$ = 160
(TABLE 10.4, Pg. 382)

$$\therefore \quad h_L = f\left[\left(100/150 \times 10^{-3}\right) + (160)\right]\left(V^2/2 \times 9.81\right) = 5.1 \text{ m}$$

THUS $5.1 = 42.1 f V^2$

$$\text{so} \quad V = \sqrt{\frac{1}{8.3 f}} \quad \leftarrow \text{A RELATIONSHIP BETWEEN } V \text{ \& } f \text{!}$$

② FIND AN EQ^{n} RELATING V AND N_R

$$N_R = \frac{VD\rho}{\mu}$$

$$= \frac{V(150 \times 10^{-3})(1000)}{1.52 \times 10^{-3}}$$

ALSO $\frac{D}{\epsilon} = \frac{150 \times 10^{-3}}{4.6 \times 10^{-5}}$

so $\frac{D}{\epsilon} = 3261$

so $N_R = (98{,}684)V$ ← A RELATIONSHIP BETWEEN V & f !

③ GUESS AT f

$f = 0.015$ (A COMMON FIRST GUESS).

④ CALCULATE V USING THE EQ^{N} OBTAINED IN STEP ①

$$V = \sqrt{\frac{1}{8.3\,f}} = \sqrt{\frac{1}{8.3 \times 0.015}}$$

$$= 2.83 \text{ m/s}$$

⑤ CALCULATE N_R USING THE EQ^{N} OBTAINED IN STEP ②

$$N_R = (98,684)V = (98,684)(2.83).$$

$$= 2.797 \times 10^{5} \quad (\therefore \text{TURBULENT}).$$

⑥ FIND A NEW AND BETTER f

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2}$$

$$f = 1.72 \times 10^{-2}$$

⑦ REPEAT ④, ⑤ AND ⑥ UNTIL f AND V STOP CHANGING.

④ $V = \sqrt{1/8.3 \times 0.0172} = 2.65$ m/s.
⑤ $N_R = (98,684)(2.65) = 2.61 \times 10^{5}$
⑥ $f = 1.73 \times 10^{-2}$.

④ $V = \sqrt{1/8.3 \times 0.0173} = 2.64$ m/s
⑤ $N_R = (98,684)(2.64) = 2.60 \times 10^{5}$
⑥ $f = 1.73 \times 10^{-2}$

④ $V = \sqrt{1/8.3 \times 0.0173} = 2.64$ m/s

NO CHANGE (TO TWO DECIMAL PLACES).

$\therefore$

STOP!

$V = 2.64$ m/s.

⑧ FIND Q

$$A = \frac{\pi D^2}{4} = \frac{\pi (150 \times 10^{-3})^2}{4} = 1.767 \times 10^{-2} \text{ m}^2$$

$$Q = VA = 2.64 \times 1.767 \times 10^{-2} = 4.66 \times 10^{-2} \text{ m}^3/\text{s}$$

$$\boxed{Q = 46.65 \text{ L/s}}$$