CLASS II - EXAMPLE

PROBLEM 11.15 M IN TEXT

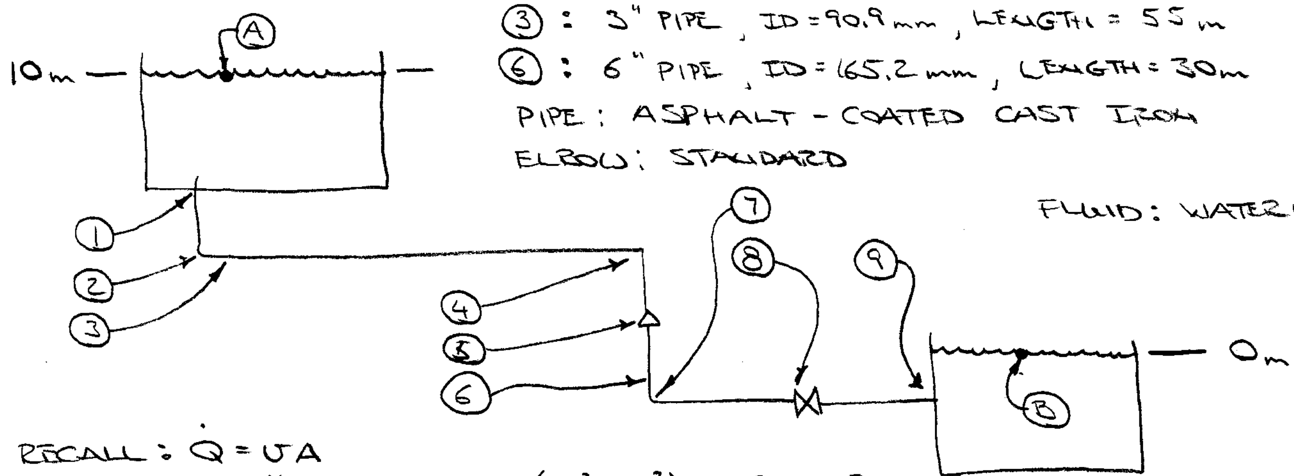
(3) : 3" PIPE, ID = 90.9 mm, LENGTH = 55 m

(6) : 6" PIPE, ID = 165.2 mm, LENGTH = 30 m

PIPE : ASPHALT - COATED CAST IRON

ELBOW : STANDARD

FLUID : WATER @ 40°C



RECALL : $\dot{Q} = vA$

$$\frac{P_A - P_B}{\gamma} + (z_A - z_B) + \frac{(v_A^2 - v_B^2)}{2g} + h_A - h_R - h_L = 0$$

LIMITS : $P_A = 0$, $z_A = 10\,m$, $v_A = 0$

$P_B = 0$, $z_B = 0\,m$, $v_B = 0$

$h_A = 0$, $h_R = 0$, h_L = SUM OF ALL h_L's = Σh_L

$\therefore (z_A - z_B) - h_L = 0$

SO $h_L = (z_A - z_B) = 10 - 0 = 10\,m$ $\boxed{h_L = 10\,m}$

(1) NOW : BECAUSE OF THE DIAMETER CHANGE AT (5) THERE ARE TWO VELOCITIES → v_3 AND v_6. BUT v_3 AND v_6 ARE RELATED BY $\dot{Q} = vA$

$Q_3 = Q_6 \rightarrow v_3 A_3 = v_6 A_6 \rightarrow v_3 = v_6 (A_6/A_3)$

AND $v_3 = 3.3\, v_6$ AND $\boxed{v_3^2 = 10.9\, v_6^2}$

NOW FOR THE FITTING AND PIPE LOSSES :

RECALL : $h_L = C_L (v^2/2g)$ AND $h_L = f\left(\frac{L_e}{D}\right)(v^2/2g)$

(1) $h_1 = 1.0\,(v_3^2/2g) = 1.0\,(10.9\, v_6^2/2g)$

(2) $h_2 = f_3 (30)(v_3^2/2g) = f_3 (30)(10.9\, v_6^2/2g)$

(3) $h_3 = f_3 (55/90.5\times10^{-3})(v_3^2/2g) = f_3 (608)(10.9\, v_6^2/2g)$

(4) $h_4 = f_3 (30)(v_3^2/2g) = f_3 (30)(10.9\, v_6^2/2g)$

(5) $h_5 = 0.45\,(v_3^2/2g) = 0.45\,(10.9\, v_6^2/2g)$

(6) $h_6 = f_6 (30/165.2\times10^{-3})(v_6^2/2g) = f_6 (182)(v_6^2/2g)$

(7) $h_7 = f_6 (30)(v_6^2/2g)$

(8) $h_8 = f_6 (40)(v_6^2/2g)$

(9) $h_9 = 1.0\,(v_6^2/2g)$

PROPERTIES

WATER @ 40°C (APP A)

$\gamma = 9.73\, kN/m^3$

$\rho = 992\, kg/m^3$

$\mu = 6.51\times10^{-4}$ Pa-s

PIPE : ASHALT - COATED CI

$\epsilon = 1.2\times10^{-4}\, m$ (T 9.1)

FITTINGS :

(1) INWARD PROJ. INLET

$C_L = 1.0$ (F 10.13)

(2)(4)(7) STD. ELBOWS

$\frac{L_e}{D} = 30$ (T 10.4)

(5) SUDDEN ENLARGEMENT

$\frac{D_2}{D_1} = \frac{165.2}{90.9} = 1.8$

ASSUME $v = 3\, m/s$

$\therefore C_L = 0.45$ (T 10.1)

(8) BUTTERFLY VALVE

$\frac{L_e}{D} = 40$ (T 10.4)

(9) EXIT LOSS

$C_L = 1.0$ (F 10.3)

FLOW AREAS

$A_3 = \frac{\pi D_3^2}{4} = 6.5\times10^{-3}\, m^2$

$A_6 = \frac{\pi D_6^2}{4} = 21.4\times10^{-3}\, m^2$

$$h_L = \Sigma h_L\text{'s} = h_1 + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9$$

$$\text{so } h_L = \left(V_6^2/2g\right)\Big[1(10.9) + f_3(30)(10.9) + f_3(608)(10.9) + f_3(30)(10.9) + 0.45(10.9) + f_6(182) + f_6(30) + f_6(40) + 1.0\Big]$$

(1)

$$= \left(V_6^2/2g\right)\Big[10.9 + 327 f_3 + 6627 f_3 + 327 f_3 + 4.9 + 182 f_6 + 30 f_6 + 40 f_6 + 1.0\Big]$$

$$h_L = \left(V_6^2/2g\right)\Big[16.8 + 7281 f_3 + 252 f_6\Big]$$

<u>BUT</u> $h_L = 10\,m$

$$\text{so ...} \quad V_6 = \sqrt{2 g h_L / [16.8 + 7281 f_3 + 252 f_6]} = \sqrt{2g(10) / [16.8 + 7281 f_3 + 252 f_6]}$$

<u>FINALLY</u> $V_6 = \sqrt{196.2 / [16.8 + 7281 f_3 + 252 f_6]}$

(2) NOW $N_R = \dfrac{V D \rho}{\mu}$ so

$$N_{R_3} = \frac{V_3 (90.9\times10^{-3})(992)}{6.51\times10^{-4}} = \underline{1.39\times10^{5}\, V_3}$$

$$\underline{N_{R_6}} = \frac{V_6 (165.2\times10^{-3})(992)}{6.51\times10^{-4}} = \underline{2.52\times10^{5}\, V_6}$$

(3) GUESS AT f : $\underline{f_3 = f_6 = 0.02}$

(4) So $V_6 = \sqrt{196.2 / [16.8 + 7281(0.02) + 252(0.02)]} = 1.08\ m/s$

AND $V_3 = 3.3\, V_6 = 3.3(1.08) = 3.56\ m/s$

(5) NOW $N_{R_3} = 1.39\times10^{5}(3.56) = 4.95\times10^{5}$

$N_{R_6} = 2.52\times10^{5}(1.08) = 2.72\times10^{5}$

(6) NEED RELATIVE ROUGHNESS : $D_3/\epsilon = 90.9\times10^{-3} / 1.2\times10^{-4} = 758$

$D_6/\epsilon = 165\times10^{-3} / 1.2\times10^{-4} = 1377$

USING MOODY'S DIAGRAM: $f_3 = 0.022$, $f_6 = 0.019$

(4) SO $V_6 = \sqrt{196.2 / [16.8 + 7281(0.022) + 252(0.019)]} = 1.04\ m/s$

$V_3 = (3.3)(1.04) = 3.43\ m/s$

(5) AND $N_{R_3} = 1.39\times10^{5}(3.43) = 4.77\times10^{5}$

$N_{R_6} = 2.52\times10^{5}(1.04) = 2.62\times10^{5}$

USE MOODY'S DIAGRAM: $f_3 = 0.022$, $f_6 = 0.019$

STOP

CONCLUSION: $V_3 = 3.43\ m/s$

$V_6 = 1.04\ m/s$

AND $\underline{Q} = VA = V_6 A_6 = (1.04)(21.4\times10^{-3}) = 2.23\times10^{-2}\ m^3/s = \underline{22.3\ L/s}$

$\boxed{Q = 22.3\ L/s}$