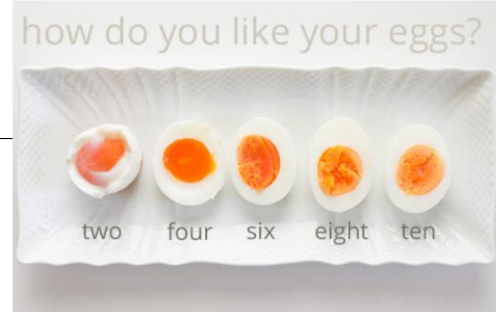
meng263 – Fluid and Heat Transfer

Transient Heat Transfer – Cooking the Perfect Egg Example



BOILING AN EGG

A TYPICAL MEDIUM SIZED EGG HAS A MASS OF 50gm AND CAN BE IDEALIZED AS A SPHERE OF WATER WITH DIAMETER OF 50 mm ...

IF IT IS TAKEN FROM THE FRIDGE AT 4°C, HOW LONG WILL IT TAKE FOR ITS INTERIOR TO REACH AN AVERAGE TEMPERATURE OF 65°C? (SAFE TO EAT, NON-RUNNY WHITE AND YOKE).

EGG: $A_s = \pi D^2 = \pi (50 \times 10^{-3})^2 = 7.854 \times 10^{-3}\ m^2$

VOLUME $= \frac{1}{6}\pi D^3 = \frac{1}{6}\pi (50 \times 10^{-3})^3 = 65.45 \times 10^{-6}\ m^3$

$\rho = 994\ kg/m^3$ (WATER AT AVG. TEMP., APPROX).

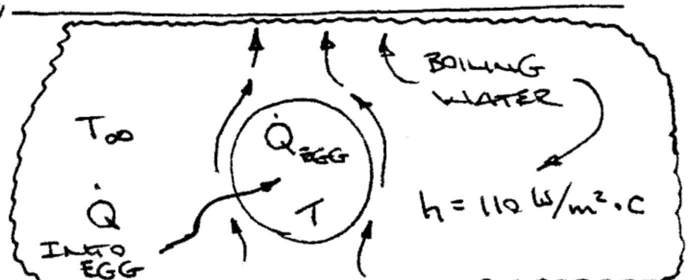
$C_p = 4178\ J/kg \cdot C$ (" " " " ")

WATER: $T = 100\ °C$ (ITS BOILING)

$\dot{Q}_{INTO\ EGG} = \dot{Q}_{STORED\ IN\ EGG}$

$hA_s(T_\infty - T)\,dT = mC_p\,dT$

SO $\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA}{mC_p}\,dT$



INTEGRATE THIS FROM $t=0$ @ $T = T_i$ = INITIAL TEMP.
TO t @ $T = T(t)$, GIVES

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{mC_p}\,t$$

NOW SET $b = \frac{hA_s}{mC_p}$

SO $\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -bt$ AND $\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$

IN OUR CASE :

$$t = \frac{\ln \frac{T(t) - T_\infty}{T_i - T_\infty}}{-b}$$

$$b = \frac{h A_s}{m C_p} = \frac{(110)(7.854 \times 10^{-3})}{(50 \times 10^{-3})(4178)} = 0.00436$$

$$= \frac{\ln\left[\frac{65 - 100}{4 - 100}\right]}{-0.00436}$$

$= 243.975$ s

$= 4.07$ MINUTES ! THE PERFECT BOILED EGG.

AMAZING THAT WORKED OUT SO WELL EH! ?

BUT IS IT ACCURATE ?

THIS CALCULATION METHOD ONLY APPLIES TO A 'LUMP SYSTEM'.

IS AN EGG A <u>LUMP SYSTEM</u> ?

CHECK ... $Bi = \frac{L_c/k}{1/h}$ IF $Bi \leq 0.1$ THEN ITS A LUMP SYSTEM.

L_c = CHARACTERISTIC LENGTH = $\frac{\text{VOLUME}}{\text{SURFACE AREA}} = \frac{65.45 \times 10^{-6} \text{ m}^3}{7.854 \times 10^{-3} \text{ m}^2} = 0.00833$ m

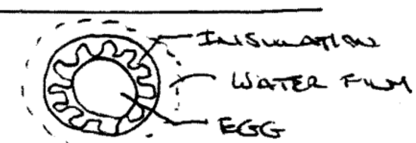
k = CONDUCTIVITY

$= 0.623$ W/m.C (LIKE WATER)

SO $Bi = \frac{[0.00833/0.623]}{[1/110]} = 1.47 > 0.1$ ∴ MY CALCS ARE TOAST!

NOTE: IF THE EGG WERE INSULATED

$Bi = \frac{L_c/k}{R}$; R = THERMAL RESISTANCE OF OUTSIDE.



IF YOU INSULATED THE EGG $Bi \leq 0.1$ IS POSSIBLE.

NOTE ALSO: $b = \frac{A_s}{m C_p R}$ (SEE TOP OF THIS PAGE)